
Zbl 041.36807
Erdős, Pál
Some asymptotic formulas in number theory. (In English)
J. Indian Math. Soc., n. Ser. 12, 75-78 (1948).

If n is a positive integer, let $f(n)$ denote the number of solutions of the equation $2^k + p = n$, k being a non-negative integer and p a prime number. The author proves that:
(I) there is a positive constant c_0 such that $f(n) > c_0 \log\log n$ for infinitely many positive integers n,
(II) if r is a fixed positive integer, there exists a positive number $c(r)$ such that $\sum_{n=1}^{x} f^r(n) < c(r)x$,
(III) if $n \equiv 7629217 \pmod{11184810}$; $f(n) = 0$.
(II) contains a theorem of *N.P.Romanoff* (Zbl 009.00801) to the effect that the positive integers expressible in the form $2^k + p$ have positive density. The author generalizes Romanoff's theorem by proving, that if $a_1 < a_2 < \cdots$ is an infinite sequence of positive integers such that $a_k \mid a_{k+1}$ for each k, then the positive integers expressible in the form $p + a_k$ have positive density if and only if there exist positive numbers $c1$ and c_2 such that $\frac{\log a_k}{k} < c_1$ and $\sum_{d/a_k} \frac{1}{d} < c_2$ for every k.
Reviewer's remark: The proof of (I) uses a result of *A.Page* (Zbl 011.14905) on the number of prime numbers in an arithmetic progression with relatively large difference. In applying this result, the author forgets to take account of a possible exceptional real primitive residue character which occurs in Page's work. This difficulty can be easily overcome in much the same manner as an analogous difficulty was handled in a joint paper of the author, the reviewer, and *S.Chowla* (Zbl 036.30702, see p. 170 of the work).

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Classification:
11N56 Rate of growth of arithmetic functions