Zbl 044.03604
Erdős, Pál
Some problems and results in elementary number theory. (In English)
Publ. Math., Debrecen 2, 103-109 (1951). [0033-3883]

Let $u_1 = 1 < u_2 < u_3 < \cdots$ be the sequence of integers of the form $x^2 + y^2$. It is immediate, as shown by Bambah and Chowla, that $u_{i+1} - u_i < cu_i^{\frac{1}{4}}$. The conjecture $u_{i+1} - u_i = o(u_i^{\frac{1}{4}})$ is still improved. Turán observed to Erdős that $u_{i+1} - u_i > c \log u_i / \log\log u_i$ for infinitely many i.
The author improves Turán's result to: $u_{i+1} - u_i > c \log u_i / (\log\log u_i)^{\frac{1}{2}}$. More generaly be proves that if $p_1 < p_2 < \cdots$ is a sequence of primes such that $\sum_{p_i \le x} \frac{1}{p} f(x) \to \infty$ as $x \to \infty$, and $v_i < v_2 < \cdots$ denote the integers wich either are not divisible by p_i or are divisible by p_i^2, then for infinitely many i $v_{i+1} - v_i > ce^{(f \log v_i)} \log v_i / \log\log v_i$. In the last part of the paper the author gives some results concerning consecutive squarefree numbers. The relations (5), (10), (11) and (28) contain some misprints.

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Classification:
11N25 Distribution of integers with specified multiplicative constraints
00A07 Problem books