Zbl 095.03902
Erdős, Pál; Hajnal, András
Some remarks on set theory. VIII. (In English)
Mich. Math. J. 7, 187-191 (1960). [0026-2285]

The authors consider independent sets and graphs (cf. also *Erdős-Fodor*, Zbl 078.04203). Let R denote the set of real numbers; for every $x \in R$ let $S(x)$ be such that $x \notin S(x) \subset R$. A subset $S \subset R$ is independent provided for every $x, y \in S$, $x \neq y$ one has $x \notin S(y)$, $y \notin S(x)$. Let H_0 denote the statement: R can be well-ordered into a Ω_c-sequence such that every set which is not cofinal with Ω_c has measure 0.
Theorem 1: If $S(x)$ $(x \in R)$ is of measure 0 and is not everywhere dense, there exist 2 real independent numbers $x \neq y$ (under H_0 there are no 3 independent real numbers).
Theorem 2: If $S(x)$ is bounded and has the exterior measure ≤ 1, then there are n independent real numbers, for every $1 < n < \omega_0$. A σ-ideal I of subsets R is said to have the property P, symbolically $I \in P$, provided it contains a transfinite sequence B_β $(\beta < \Omega_c)$ of members such that every member of I is contained in some B_β.
Theorem 3: If $\aleph_1 = c$ and $I \in P$, then each graph G_R on R contains an infinite chain or an antichain that is not in I (the statement may not hold provided $I \notin P$).
Theorem 5: Let $m < c$. Let I_α $(\alpha < \Omega_c)$ be a sequence of σ-ideals of subsets of R, each with property P. Then every graph G_R contains, for every $n < \omega$, a subgraph $\{x_i\} \cup \{y_\nu\}$ $(1 < i \leq n,\ 1 < \alpha < \Omega_c)$ such that (x_i, y_α) is connected or there is an antichain in G_R which is contained in no I_α. The authors ask whether theorem 5 holds for $m = c$; they conjecture also that theorem 5 may not hold if the property P is delated, even for $n = m = 2$.

G.Kurepa

Classification:
04A99 Miscellaneous topics in set theory