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On a classification of denumerable order types and an application to the partition calculus (In English)

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Let Θ and Θ' be denumerable order types; Θ is discrete if $\eta \not\leq \Theta$. It is shown that if Θ is discrete, Θ has a rank $\varrho(\Theta)$ (an ordinal $< \omega_1$) defined from the way Θ is attainable from 0 and 1 via a transfinite process of ω- and ω^*-additions. It is shown that if Θ is not discrete, Θ is a sum of type η, $1+\eta$, $\eta+1$, or $1+\eta+1$ of non-zero discrete types. Among the theorems (here paraphrased) in the partition calculus proved by using rank are the following statements (the bracketed insertions have been made by the reviewer). $\Theta \to (\Theta, \aleph_0)^2$ if (and only if) $\Theta = \omega$ or $\Theta = \omega^*$ or $\eta \leq \Theta$ [or $\Theta < 2$]. $\Theta \not\to (\Theta', \aleph_0)^2$ if Θ is discrete and $\Theta' \neq n + \omega^*$ and $\Theta' \neq \omega + n$ for each $n < \omega$ [and Θ' is infinite]. $\Theta \to (\omega + n, \aleph_0)^2$ if and only if $\omega \cdot \omega^* \leq \Theta$.

[Minor errors: On line 27 of p. 125 replace "$\overline{\overline{S'' \cdot S_{n'_0}}} = \aleph_0$" by "either both $n_0 < n'_0$ and $\overline{\overline{S'' \cdot S_{n'_0}}} = \aleph$". Lines 18-20 of p. 125 neglect the possibility that $\overline{\overline{S'}} = \aleph_0$ and $[S']^2 \subset I_2$; however, this possibility may be handled trivially.]

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Classification:
05D10 Ramsey theory
04A10 Ordinal and cardinal numbers; generalizations
04A20 Combinatorial set theory
03E05 Combinatorial set theory (logic)