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On the distribution of divisors of integers in the residue classes (mod d) (In English)

Bull. Soc. Math. Grece, N. Ser. 6, No.1, 27-36 (1965).

Let k and l denote integers satisfying $0 < l < k, (l,k) = 1$. Denote by $f(x;k,l)$ the number of positive integers less than x which have a divisor congrent to l (mod k), by $F(x;k)$ the number of positive integers less than x which have a divisor congruent to l (mod k) for every l, by $Q(x)$ the number of positive integers less than x which have no divisor of the form $p(kp+1)$, by $d(n;k,l)$ the number of divisors of n which are congruent to l (mod k). The author proves or outlines the proof of the following theorems: 1. Let $\varepsilon > 0$ be fixed but arbitrary, $k < 2^{(1-\varepsilon)\log\log x}$. Then uniformly in $kF(x;k) = x + o(x)$. 2. Let $\varepsilon > 0$ be fixed but arbitrary, $k > 2^{(1+\varepsilon)\log\log x}$. Then uniformly in k and l $f(x;k,l) = x/l + o(x)$. 3. $Q(x) = (1+o(1))e^{-c}x/\log 2\log\log x$ where c is Euler's constant. 4. Let $\varepsilon > 0$ be fixed but arbitrary $k < 2^{[(1-\varepsilon)\log\log x]/2}$. Then for every $\eta > 0$ we have for every l_1 and l_2, for all but $o(x)$ integers less than x

$$1-\eta < d(n;k,l_1)/d(n;k,l_2) < 1+\eta.$$

The proofs of Theorems 1 and 4 are based on recent group theoretic results of the author and A. Rényi [J. Analyse math. 14, 127-138(1965)], the theorem of Siegel and Walfisz on the distribution of primes in an arithmetic progression, and a theory of Hardy, Ramanujan, and Turan on the number of prime factors of n. The proofs of Theorems 2 and 3 are outlined only.

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11N69 Distribution of integers in special residue classes