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Erdős, Pál; Hajnal, András
On a combinatorial problem (In Hungarian)
Mat. Lapok 19, 345-348 (1968). [0025-519X]

Let S be a set of n elements. Let $f(A)$ be a set function which makes correspond to every subset A of S an element of $S-A$. Put $F(A)=\bigcup_{B\subset A} f(B)$ where B runs through all subsets of A. Let $H(n)$be the smallest integer for which there is a function f so that for every $S_1\subset S$, $|S_1|\geq H(n)$ we have $F(S)=S$. We prove

$$\log n/\log 2<H(n)<\log n/\log 2+3\log\log n/\log 2+o(\log\log n).$$

We conjecture $\lim_{n=\infty}(H(n)-\log n/\log 2)=\infty$ but cannot even prove $H(n)>\log n/\log 2+1$.

Classification:
05D05 Extremal set theory
04A20 Combinatorial set theory