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On some problems involving inaccessible cardinals (In English)
Essays Found. Math., dedicat. to A.A. Fraenkel on his 70th Anniv. 50-82 (1962).

[For the entire collection see Zbl 128.24103.]
In this paper, several properties of infinite cardinals are investigated. Let $P_1, \dots, P_4, Q, R$ be the following properties: $P_1(\lambda)$: There is a set with power λ which is simply ordered by a relation $\leq$ in such a way that no subset of it with power λ is well ordered by the same relation $\leq$ or by the converse relation $\geq$. $P_2(\lambda)$: There is a complete graph on a set of power λ that can be divided in two subgraphs neither of which includes a complete subgraph on a set of power λ. $P_3(\lambda)$: Every λ-complete prime ideal in the set algebra formed by all subsets of λ is principal. $P_4(\lambda)$: There is a λ- complete and λ-distributive Boolean algebra which is not isomorphic to any λ-complete set algebra. $Q(\lambda)$: There is a ramification system $\langle A, \leq\rangle$ of order λ such that (1) the set of all elements $x \in A$ of order ξ has power $< \lambda$ for every λ, (2) every subset of A well-ordered by $\leq$ has power λ. $R(\lambda)$: there is λ-distributive Boolean algebra which is λ-generated by a set of power λ and which is not isomorphic to any λ-complete set algebra.
By ${S_1}^D \to S_2$ we mean that for every infinite cardinal in D property S_1 implies S_2. And let C, AC, SL, IC be the class of all infinite, accessible, singular strong limit and inaccessible cardinals, respectively. We write $S_1 \to S_2$ instead of ${S_1}^C \to S_2$.
The main result of this paper is a diagram of implications. It is also obtained that the property R applies a very comprehensive class of inaccessible cardinals. Most of implications in the opposite direction, and the existence of an inaccessible cardinal which does not have the property R still remain open.

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Classification:
03E55 Large cardinals