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Erdős, Paul; Straus, E.G.
Nonaveraging sets. II (In English)
Combinat. Theory Appl., Colloquia Math. Soc. Janos Bolyai 4, 405-411 (1970).
[For the entire collection see Zbl 205.00201.]

We wish to consider sets of integers $A = \{a_1, \dots, a_n\}$ so that $0 \leq a_1 < a_2 < \dots < a_n \leq x$ and no a_i is the arithmetic mean of any subset of A consisting of two or more elements. In Part I [by the second author in Proc. Sympos. pure Math., Am. Math. Soc. (1967)] is initiated the study of the maximal number of elements in nonaveraging sets and sets which satisfy related conditions. Using the notation of Part I we define $f(x)$ as the maximal number of elements in a nonaveraging set; $h(x)$ as the number of elements of a maximal set of integers in the interval $[0, x]$ such that no two distinct subsets have the same arithmetic mean; and $h^*(x)$ as the number of elements of a maximal set of integers in $[0, x]$ such that no two subsets with a relatively prime number of elements have the same arithmetic mean. In Part I we proved ($\log_r x = \log x / \log r$):

$$\log_2 f(x) > \sqrt{2 \log_2 x} + \frac{1}{2} + O(1/\sqrt{\log 2}),$$

$$(*) \qquad (1 + \sigma(1)) \log x / \log\log x < h(x) < \log_2 x + O(\log\log x),$$

$$\log_2 h^*(x) \geq \sqrt{\log_2 x} - 1 + 0(1/\sqrt{\log x}).$$

In the present note we prove in §2 that (*) can be replaced by

$$-1 + \log_4 x \leq h(x) < \log_2 x + O(\log\log x).$$

Next, in §3, we prove that even if we ease the restriction on our sets so that only subsets with different numbers of elements must have different averages then the maximal number, $h^{**}(x)$, of elements satisfies

$$h^{**}(x) < c(\log x)^2 \text{ for some constant } c.$$

Finally in §4 we get an upper bound for $f(x) < cx^{3/4}$.

Classification:
05A99 Classical combinatorial problems