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Erdős, Paul; Renyi, Alfréd
On random entire functions. (In English)
Zastosowania Mat. 10, 47-55 (1969).

Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ be an arbitrary entire function, held fixed in all that follows. For $r > 0$ let $M(r) = \max(|f(z)| : |z| = r)$ be the maximum modulus function of f and $\mu(r) = \max(|a_n|r^n : n \geq 0)$ the maximum term in the series expansion of f. The following extension of Wiman's theorem was proved by Rosenbloom: for every $\delta > 0$ there exists a subset E_δ of fintie logarithmic measure such that if $r \notin E_\delta$, then

$$M(r) < \mu(r)[\log \mu(r)]^{1/2}[\log \log \mu(r)]^{1+\delta}.$$

For $0 \leq t < 1$ let $R_n(t) = \operatorname{sign} \sin(2^n \pi t)$ denote the n-th Rademacher function, $n \geq 0$. The present paper considers the class of entire functions obtained by giving random signs to the terms in the series expansion of f above; explicitly, the entire functions which can be written $f(z,t) = \sum_{n=0}^{\infty} a_n R_n(t) z^n$, $0 \leq t < 1$. Keeping the notation above, let $M(r,t) = \max(|f(z,t)| : |z| = r)$. The main result is: for every $\delta > 0$ and almost all $t \in [0,1)$, there exists a subset $E_\delta(t) \subset R_+$ of finite logarithmic measure (depending on t) such that for $r \notin E_\delta(t)$,

$$M(r,t) < \mu(r)[\log \mu(r)]^{1/4}[\log \log \mu(r)]^{1+\delta}.$$

Two related results are also given.

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Classification:
30D20 General theory of entire functions
60-XX Probability theory and stochastic processes