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Ordinary partition relations for ordinal numbers. (In English)

Period. Math. Hung. 1, 171-185 (1971). [0031-5303]

In this paper a number of positive and negative partition relations of the form $\alpha \to (\beta,\gamma)^2$ are established [see *P. Erdős* and *R. Rado*, Bull. Am. Math. Soc. 62, 427-489 (1956; Zbl 071.05105)]. *E. Specker* [Commentarii Math. Helvet. 31, 302-314 (1957; Zbl 080.03703)] proved that $\omega^2 \to (\omega^2, m)^2$ for $m < \omega$ and *A. Hajnal* [Proc. Natl. Acad. Sci. USA 68, 142-144 (1971; Zbl 215.05201)], proved that the corresponding result for higher cardinals fails, by showing that, if $\aleph_\zeta$ is regular and GCH is assumed, then $\omega^2_{\zeta+1} \nrightarrow (\omega^2_{\zeta+1}, 3)^2$.

This negative result is extended here and some complementary positive theorems are proved. Thus, if $\aleph_\zeta$ is regular, GCH is assumed, $k, t < \omega$, $m = (t+1)(k+1)$ and $\mu < \beta = \omega^{k+2}_{\zeta+1}$, then $\omega^m_{\zeta+1} \nrightarrow (\beta, t+2)^2$, $\omega^m_{\zeta+1} \to (\mu, t+2)^2$, $\omega^{m+1}_{\zeta+1} \nrightarrow (\beta+1, t+2)^2$ and $\omega^{m+1}_{\zeta+1} \to (\beta, t+2)^2$. This leaves some questions open.

For example, the authors ask if $\omega^2_1 \to (\omega_1\tau, 4)^2$ for all $\tau < \omega_1$. *C. C. Chang* [J. Comb. Theory, Ser. A 12, 396-452 (1972; Zbl 266.04003)], proved that $\omega^\omega \to (\omega^\omega, 3)^2$ (and it is known that $\omega^\omega \to (\omega^\omega, m)^2$ for all $m < \omega$). In contrast to this, it is shown here (assuming GCH) that $\sigma \nrightarrow (\omega^\omega_1, 3)^2$ for all $\sigma < \omega_2$. It is not known if the analogous result $\sigma \nrightarrow (\omega^{\omega_1}_2, 3)^2$ holds for all $\sigma < \omega_3$.

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Classification:

04A20 Combinatorial set theory

05A17 Partitions of integres (combinatorics)