Zbl 272.10021
Erdős, Paul; Hall, R.R.
Some distribution problems concerning the divisors of integers. (In English)
Acta Arith. 26, 175-188 (1974). [0065-1036]

The paper is concerned with the distribution (mod 1) of the numbers $\log d$, where d runs through the divisors of an integer n. Let $|x|$ denote the distance from x to the nearest integer. Then the authors' main result is the following: let α and c be real numbers. The integers n having a divisor d satisfying

$$0 < |\log d - \alpha| < 2^{-\log\log n - c\sqrt{\log\log n}} \tag{1}$$

have asymptotic density $\delta(c) = (2\pi)^{-1/2}\int_c^\infty e^{-u^2/2}du$, moreover if c is replaced by a function of n tending to $+\infty$ or $-\infty$, the density is 0 or 1 respectively. If there exists an integer $m(\alpha)$, necessarily unique, such that $\log m(\alpha) \equiv \alpha$ (mod 1), the density is increased if equality is allowed on the extreme left of (1): it becomes $\delta + (1-\delta)/m(\alpha)$ where $\delta = \delta(c)$.

Classification:
11J71 Distribution modulo one
11B83 Special sequences of integers and polynomials