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Partition relations for η_α and for $\aleph_\alpha$-saturated models. (In English)

Theory Sets Topology, Collection Papers Honour Felix Hausdorff, 95-108 (1972).

[For the entire collection see Zbl 256.00006.]

Let R_α be the set of all non-zero dyadic sequences $(x_\nu; \nu < \omega_alpha)$ which are eventually zero, this is, there is some $\nu < \omega_\alpha$ such that $x_\nu = 1$ and $x_\mu = 0$ for $\mu > \nu$. The order type of R_α under the lexicographic ordering is denoted by η_α. The following two theorems are proved (under the assumption of the Generalized Continuum Hypothesis): I. if $\aleph_\alpha$ is regular and m is a cardinal with $m < \aleph_alpha$ then $\eta_\alpha \to (\eta_\alpha, [m, \eta_\alpha])^2$. For all β, $\eta_{\beta+1} \to (\eta_{\beta+1}[\eta_\beta, \eta_\beta])^2$. The symbol $\eta_\alpha \to (\eta_\alpha, [m, \eta_\alpha])^2$ means the following: Whenever the set of unordered pairs of elements from R_α is partitioned into two classes either there is a subset X of R_α of order type η_α all the pairs from which lie in the first class, or else there are subsets M, N of R_α where M has power m and N has order type η_α such that all the pairs $\{x, y\}$ where $x \in M$, $y \in N$, $x \neq y$ fall into the second class. The second partition symbol has an analogous meaning. In fact, results more general than these are established. These are applied to show, in particular, that results corresponding to I and II hold if the set R_α of order type η_α is replaced by certain $\aleph_\alpha$- saturated models. Other partition relations for η_α-sets have appeared in an earlier paper of *P. Erdős, E. C. Milner* and *R. Rado* [J. London math. Soc., II. Ser. 3, 193-204 (1971; Zbl 212.02204)].

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Classification:
04A20 Combinatorial set theory
03C99 Model theory (logic)
04A10 Ordinal and cardinal numbers; generalizations