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Problems and results on 3-chromatic hypergraphs and some related questions. (In English)

Infinite finite Sets, Colloq. Honour Paul Erdős, Keszthely 1973, Colloq. Math. Soc. Janos Bolyai 10, 609-627 (1975).

[For the entire collection see Zbl 293.00009.]

The authors investigate several extremal problems on set systems and state many unsolved problems. In this short review I can state only two of them. Denote by $f(n)$ the smallest integer for which there is a family $\{A_k\}$ of subsets, $|A_k| = n$, $|A_{k_1} \cap A_{k_2}| \leq 1$; $1 \leq k \leq f(n)$ and which is three-chromatic, i.e. if $\varphi \cap A_k \neq \emptyset$ for every $1 \leq k \leq f(n)$. Then for some $k \varphi \supset A_k$. We prove

$$c_1 4^n / n^4 < f(n) < c_2 4^n n^4. \tag{1}$$

It would be desirable to have an asymptotic formula for $f(n)$. Let $g(n)$ be the smallest integer for which there is a family $\{A_k\}$, $1 \leq k \leq g(n)$, $|A_k| = n$, $A_{k_1} \cap A_k \neq \emptyset$ so that for any $|\varphi| = n - 1$ there is an A_k with $\varphi \cap A_k = \emptyset$. Is it true that $g(n)/n \to \infty$? We only get crude upper and lower bounds for $g(n)$.

Classification:
05C35 Extremal problems (graph theory)
05C99 Graph theory
05C15 Chromatic theory of graphs and maps
00A07 Problem books