
Zbl 328.05123

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On sparse graphs with dense long paths. (In English)

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An acyclic directed graph G is said to have property $P(m,n)$ if for any set X of m vertices of G, there is a directed path of length n in G which does not intersect X. Let $f(m,n)$ denote the minimum number of edges a graph with property $P(m,n)$ can have. (Hereafter, $c_1, c_2, \ldots$ denote suitable positive constants.) Theorem. $c_1 n \log n / \log\log n < f(n,n) < c_2 n \log n$. The graph constructed in order to establish the upper bound on $f(n,n)$ has $c_3 n$ vertices. In this case, the upper bound is essentially best possible since it is shown that for c_4 sufficiently large, if a graph on $c_4 n$ vertices has property $P(n,n)$ then it must have at least $c_5 n \log n$ edges.

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Classification:
05C35 Extremal problems (graph theory)
05C20 Directed graphs (digraphs)