Zbl 328.54017
Erdős, Paul; Rudin, Mary Ellen
A non-normal box product. (In English)
Infinite finite Sets, Colloq. Honour Paul Erdős, Keszthely 1973, Colloq. Math. Soc. Janos Bolyai 10, 629-631 (1975).
[For the entire collection see Zbl 293.00009.]
The box product of the family $\{X_n \mid n \in \omega\}$ of topological spaces is just $\prod_{n\in\omega} X_n$ with a basis consisting of arbitrary products of open sets. The paper concerns families where each X_n is an ordinal with the order topology. A subset F of ω^ω is a κ-scale if i) $\{^{F=\{f_\alpha|\alpha<\kappa\}}\}$, ii) $\alpha < \beta < \kappa$ implies $f_\alpha(n) < f_\beta(n)$ for all but finitely many n and iii) for any $f \in \omega^\omega$ there are an $\alpha < \kappa$ and an $m < \omega$ with $f(m) < f_\alpha(m)\ \forall m > n$. The following is proven. Theorem. If $\kappa \neq \omega_1$ is the minimal cardinality of a scale then $\prod X_n$ is not normal where $X_0 = \kappa$ and $X_n = \omega + 1$ all $n > 0$. The paper also states a number of facts about such spaces.

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Classification:
54D15 Higher separation axioms
03E15 Descriptive set theory (logic)
04A15 Descriptive set theory
54A25 Cardinality properties of topological spaces
54G15 Pathological spaces