Zbl 347.41009
Erdős, Paul; Reddy, A.R.
Rational approximation on the positive real axis. (In English)
Proc. Lond. Math. Soc., III. Ser. 31, 439-456 (1975); corrigendum ibid. 35, 290 (1977). [0024-6115]

This paper is a continuation of the authors' researches on the problem of approximating the reciprocal of an entire function with nonnegative Taylor coefficients in the uniform norm on the positive half-axis by means of reciprocals of polynomials. [previous papers Bull. Amer. math. Soc. 79, 992-993 (1973; Zbl 272.41007) and Period. math. Hungar 6, 241-244 (1975; Zbl 273.41012), ibid. 7, 27-35 (1976; Zbl 337.41020)]. Quoting the authors this paper may serve as a guide to those interested in this topic. From the results we quote: Letting $\lambda_{0,n}$ denote the degree of approximation with n^{th} degree polynomials in the problem described above, we have: For any $\epsilon > 0$ and $k \geq 1$ there exist infinitely many n such that

$$\lambda_{0,n} \leq \exp(-n/\log n \log\log n \dots (\log^{(k)} n)^{1+\epsilon})$$

but there exists to every k and every large c a function such that

$$\lambda_{0,n} \geq \exp(-cn/\log n \log\log n \log^{(k)} n).$$

For an entire function of order ρ, $0 < \rho < \infty$, type τ and lower type ω we have

$$\limsup \lambda_{0,n}^{\rho/n} \leq \exp(-\omega/(e+1)\tau).$$

The paper also contains lower estimates for the above case, some estimates for functions of zero order, and a number of examples.

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Classification:
41A20 Approximation by rational functions
30E10 Approximation in the complex domain