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Rational approximation. II. (In English)

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Let π_m denote the class of all real polynomials of degree at most m and $\pi_{m,n}$ denote the collection of all rational functions $r_{m,n}(x) = \frac{p(x)}{q(x)}, p \in \pi_m, q \in \pi_n$. Let $\lambda_{m,n} \equiv \lambda_{m,n}(f^{-1}) = \inf_{r_{m,n}\in\pi_{m,n}} \left\| \frac{1}{f(x)} - \gamma_{m,n}(x) \right\|_{l_\infty[0,\infty]}$ where f, given by $f(z) = \sum_{k=0}^{\infty} a_k z^k$, is an entire function with all non-negative coefficients. In Part I [*P.Erdős* and *A.R.Reddy*, Adv. Math. 21, 78-109 (1976; Zbl 334.00019)] , the authors mainly reviewed and proved certain results concerning $\lambda_{0,n}$. In the present paper, which contains 22 theorems, the authors devote themselves to show that for certain classes of entire functions the error obtained by rational functions of degree n in approximating on $[0,\infty)$ under the uniform norm is much smaller than the error obtained by recipocals of polynomials of degree n. For example, they show in Theorem 10 that if f is an entire function of order $\rho(1 \leq \rho < \infty)$, type τ and lower type $\omega(0 < \omega \leq \tau < \infty)$, then for every polynomial $P_n(x)$ of degree n and all large n, there exist positive constants a and b for which $\left\| \frac{x+1}{f(x)} - \frac{1}{P_n(x)} \right\|_{L_\infty[0,\infty)} \geq a \exp((-bn^{1-1/3\rho})$ whereas in Theorem 17 they establish that for such functions there is some $\beta(0 < \beta < 1)$ such that $\lambda_{1,n}\left(\frac{1+x}{f(x)}\right) \leq \beta^n$. It has also been shown that for certain entire functions, for example $f(z) = e^{e^z}$, there is little difference between the errors abtained by rational functions and the errors obtained by recipocals of polynomials. Incidentally, the following interesting results has also been obtained:

$$\lim_{n\to\infty} \left[\lambda_{0,n}(xe^{-x})\right]^{1/2\log n} = e^{-1} = \lim_{n\to\infty} \left[\lambda_{0,n}((1+x)e^{-x})\right]^{1/(2n)^{2/3}}.$$

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Classification:
30E10 Approximation in the complex domain
41A20 Approximation by rational functions
41A25 Degree of approximation, etc.