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De Koninck, J.-M.; Erdős, Paul; Ivić, A.
Reciprocals of certain large additive functions. (In English)
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Let $\beta(n)$ be the sum of distinct prime divisor of n and $B(n)$ the sum of all prime factors of n (counting multiples). Methods are used that can be as well applied to several pairs of similarly related large additive functions, to prove three theorems. Theorem 1 gives upper and lower bounds on $\sum_{2\le n\le x} 1/\beta(n)$ and $\sum_{2\le n\le x} 1/B(n)$. Theorem 2 estimates $\sum_{2\le n\le x} \beta(n)/B(n)$ and $\sum_{2\le n\le x} B(n)/\beta(n)$ in the form of $x + O(x\exp(-C(\log x\log\log x)^{\frac{1}{2}}))$. Theorem 3 estimates $\sum'_{n\le x} 1/(B(n)-\beta(n))$ as $Cx + O(x^{\frac{1}{2}}\log x)$. The constant C in theorem 3 is given explicitly and $\sum'$ denotes the sum over those n for which $B(n)\ne\beta(n)$. Most of the method is elementary but an analytical method using the Riemann zeta-function is involved in the proof of theorem 3.

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