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On sums of Rudin-Shapiro coefficients. II. (In English)
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This paper is an extension of previous work by the first and third authors on the Rudin-Shapiro sums $s(x)=\sum_{k=0}^{[x]} a(k)$, where $a(k)$ is defined to be plus of minus on according as the number of pairs of consecutive 1's in the binary represantation of k is even or odd. [See Ill. J. Math. 22, 126-148 (1978; Zbl 371.10009).] The properties of these sums are developed further by introducing the limit function

$$\lambda(x)=\lim_{k\to\infty}(s(4^k x)/\sqrt{a^k x}), x>0,$$

which turn out to be a continuous function from $(0,\infty)$ onto the interval $\left[\sqrt{(3/5)},\sqrt{6}\right]$ and which satisfies the equation $\lambda(4x)=\lambda(x)$. this function is used to represent $s(x)$ as a logarithmic Fourier series:

$$s(x)=\sqrt{x}\sum_{n=-\infty}^{\infty} c_n x^{\pi n/\log 2}+a(x), x>0,$$

Where $a(x)$ is an explicit bounded function of the digits of x to the base 4, which extends $a(k)$ to the set of positive reals. The series (1) is shown to converge for almost all positive real numbers; in particular, it converges for all $x>0$ which are normal to the base 4. It turns out that $\lambda(x)$ is non-differentiable on this same set. This is then used to show that the Dirichlet series $\eta(\tau)=\sum_{n=1}^{\infty} a(n)n^{-\tau}$ has a meromorphic continuation to the whole complex plane with infinitely many poles. Finally, $\lambda(x)$ is used to prove that the sequence $\left\{\frac{s(n)}{\sqrt{n}}\right\}_{n\geq 1}$ has a logarithmic sistribution function on the interval $\left[\sqrt{(3/5)},\sqrt{6}\right]$, but that the cumulative distribution function to this sequence does not exist.

Classification:
11B83 Special sequences of integers and polynomials
11K65 Arithmetic functions (probabilistic number theory)
11K16 Normal numbers, etc.
11A63 Radix representation

Keywords:
Rudin-Shapiro sums; binary representations; Fourier series; Dirichlet series; logarithmic distribution