Zbl 473.10004
Erdős, Pál
On prime factors of binomial coefficients. (In Hungarian)
Mat. Lapok 28, 287-296 (1980). [0025-519X]

Denote $P(m)$ the greatest, while $p(m)$ the smallest prime divisor of the natural number m. Let $\binom{n}{k} = UVW$, where $P(U) \leq k$, $P(V) > k$, but $P(V) \leq n-k$, and $W = \prod_{n-k<p\leq n} p$. This paper deals with $M = \max\{U; V; W\}$, assuming $n \geq 2k$. The following results are proven. For $k > 3$, $M = U$ holds only for finitely many pairs n, k. Let $\delta > 0$. If $k \geq k_0(\delta)$ and $n < (14/3 - \delta)k$ then $M = W$, but if $n > (14/3 + \delta)k$ then $M = V$. Furthermore if $k \geq 10$ or $k = 6, 8$ and $n > n_0(k)$ then $M = V$. Finally for any integer r there exist infinitely many pairs n, k such that $U(n,k) > W(n,k) > n^r$. The results are ineffective, because in the proofs an ineffective result of K. Mahler concerning the greatest prime divisor of binomial coefficients is used. In the paper one can also find some questions and problems.

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Classification:
11A05 Multiplicative structure of the integers
05A10 Combinatorial functions
11A41 Elemementary prime number theory

Keywords:
smallest prime divisor; greatest prime divisor; ineffective results