Zbl 476.10045
Erdős, Pál; Szemeredi, Endre
Remarks on a problem of the American Mathematical Monthly. (In Hungarian)
Mat. Lapok 28, 121-124 (1980). [0025-519X]

Let $A = a_1 < a_2 < \dots$ be a sequence of positive integers. Let $F(A, x, i)$ denote the number of k's for which the least common multiple $[a_k, a_{k+1}, \dots, a_{k+i-1}]$ satisfies in the inequality $[a_k, a_{k+1}, \dots, a_{k+i-1}] \le x$. Some years ago *P.Erdős* formulated the problem in Am. Math. Mon. to prove that $F(A, x, i) < c_i x^{1/i}$, where c_i is a constant depending only on i. This statement is false. This can be seen from the following results of the paper (see III).
I. For any A we have

$$\limsup_{x\to\infty} \frac{F(A,x,2)}{\sqrt{x}} \le \sum_{k=1}^{\infty} \frac{k^{1/2} - (k-1)^{1/2}}{k}$$

and

$$\liminf_{x\to\infty} \frac{F(A,x,2)}{\sqrt{x}} = 0 \tag{1}$$

provided that in (1) the sign=holds.
II. For any A we have $\liminf_{x\to\infty} \frac{F(A,x,2)}{\sqrt{x}} \le \frac{1}{2}$.
III. If $i > 4$, then there exists an $\alpha_i > 0$ such that for each sufficiently large x and suitable A we have

$$F(A, x, i) > x^{\frac{1}{i} + \alpha_i}.$$

The authors conjecture that III holds for $i = 4$, too. For $i = 3$ they have proved that for each sufficiently large x and any A $F(A, x, 3) < c_0 x^{1/3} \log x (c_0 > 0)$ and that there is such an A that $F(A, x, 3) > c_1 x^{1/3} \log x (c_1 > 0)$ for infinitely many x holds. The question whether there is such an A that for each x the inequality $F(A, x, 3) > c_2 x^{1/3} \log x (c_2)$ holds, remain open.

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Classification:
11B83 Special sequences of integers and polynomials
11A05 Multiplicative structure of the integers

Keywords:
sequence of positive integers; least common multiple