**Zbl 499.41004**
**Erdős, Pál; Vertesi, Peter**
*On the Lebesgue function of interpolation.* (In English)
**Proc. 13th Symp. Ring theory, Okayama 1980, 299-309 (1981).**

[For the entire collection see Zbl 436.00014.]

One consider a triangular matrix $Z = \{x_{k,n}\}$ $(n \in \mathcal{N}, k = 1(1)n)$ of distinct real arbitrary nodes, such that: $1 \leq x_{n,n} < x_{n-i,n} < \cdots < x_{1,n} \leq 1$. Let $\ell_k(x) = \ell_{k,n}(Z, x)$ to be corresponding fundamental polynomials of the Lagrange interpolation. It is known that the Lebesgue function and the Lebesgue constant, defined respectively by $\lambda_n(x) = \lambda_n(Z, x) = \sum_{k=1}^n |\ell_k(x)|, \lambda_n = \lambda_n(Z) = \max \lambda_n(x)$ for $-1 \leq x \leq 1$, play a decisive role in the convergence and divergence properties of Lagrange interpolation. In 1961
P.Erdős [Acta Math. Acad. Sci. Hung. 12, 235-244 (1961; Zbl 098.04103)] has proved that for any system of nodes $x_{k,n}$ $(k = 1(1)n)$ we have $\lambda_n > 2\pi^{-1}\ell nn - c(n \geq n_0)$, where $c$ is a certain positive absolute constant. In this paper the authors prove the following remarkable theorem: If $\varepsilon$ is any given positive number, then for arbitrary matrix $Z$ there exist sets $H_n$, with $|H_n| \leq \varepsilon$ and $\eta(\varepsilon) > 0$, such that $\lambda_n(x) > \eta(\varepsilon)\ell nn$, whenever $x \in [-1, 1]|H_n$ and $n \geq n_0(\varepsilon)$. The case of Chebyshev nodes showsthat this order is best possible. In the proof of this theorem the authors use some results from their recent common paper [ibid, 36, 71-89 (1980; Zbl 463.41002)]. Finally we mention the following important corollary of this theorem: Let $\varepsilon > 0$ and $\eta(\varepsilon) > 0$ be as above. If $S_n \subseteq [-1, 1]$ are arbitrary measurable sets then for any matrix $Z$ we have $\int_{S_n} \lambda_n(x)dx > (|S_n| - \varepsilon)\eta(\varepsilon)\ell nn$, whenever $n \geq n_0(\varepsilon)$. The special case $S_n = S = [a, b]$ has been investigated earlier by *P.Erdős* and *J.Szabados* [ibid. 32, 191- 195 (1978; Zbl 391.41003)].

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Classification:
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41A17 Inequalities in approximation
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