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On the decomposition of graphs into complete bipartite subgraphs. (In English)

Studies in pure mathematics, Mem. of P. Turán, 95-101 (1983).

[For the entire collection see Zbl 512.00007.]

A B-covering (respectively B-decomposition) of a graph G is a collection of complete bipartite graphs G_i such that any edge of G is in at least (respectively exactly) one $G_i (i = 1, 2, ..., t)$. Let $\beta(G; B)$ (respectively $\alpha(G; B)$) denote the minimum value of $\sum_{i=1}^{t} |V(G_i)|$ over all B- coverings (respectively B-decompositions) of G. Let $\beta(n; B)$ (respectively $\alpha(n; B)$) denote the maximum value of $\beta(G; B)$ (respectively $\alpha(G; B)$) as G ranges over all graphs on n vertices. "In this paper we show that, for any positive ϵ, we have

$$(1-\epsilon)\frac{n^2}{2e\log n} < \beta(n; B) \leq \alpha(n; B) < (1+\epsilon)\frac{n^2}{2\log n},$$

where e is the base of the natural logarithms, provided n is sufficiently large." A number of related questions and conjectures are discussed. For example, if G_n denote the set of the $2^{\binom{n}{2}}$ labelled graphs on n vertices, it is conjectured that

$$\lim_{n\to\infty} \sum_{G\in G_n} \alpha(n; B)/2^{\binom{n}{2}} n^2/\log n$$

exists.

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Classification:
05C35 Extremal problems (graph theory)
05C99 Graph theory
60C05 Combinatorial probability

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