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*Quantitative forms of a theorem of Hilbert.* (In English)

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For positive integers $m$, $a$ and $a_k$, $1 \leq k \leq m$ define an m- cube $Q_m$ to be the set $\{a + \sum_{k=1}^{m} \epsilon_k a_k : \epsilon_k = 0 \text{ or } 1,\ 1 \leq k \leq m\}$. Hilbert proved that for any positive integers $m$ and $r$ there exists a least integer $h(m,r)$ such that if the set { 1,2,...,h(m,r)} is arbitrarily partitioned into r classes $C_k$, $1 \leq k \leq r$, some $C_i$ must contain an m cube. Schur proved that for any $r$, there is an $s(r)$ so that in any partition of { 1,2,...,s(r)} into $r$ classes some class contains a projective 2- cube $Q_2^*(a, a_1, a_2) - \{0\}$ with $a = 0$. This was extended by Rado for projective m-cubes and further extended by Hindman to infinite projective cubes i.e. for $\{\sum_{k=1}^{\infty} \epsilon_k a_k : \epsilon_k = 0 \text{ or } 1 \text{ with } 0 \leq \sum_{k=1}^{\infty} \epsilon_k < \infty\}$.
In this article the authors have investigated the function $h(m,r)$ and several related ones. For the first interesting case $m = 2$ it is proved that $H(2,r) = (1 + 0(1))r^2$. This result is closely related to Ramsey numbers for 4- cycles. Bounds are also obtained for deleted 2-cubes.

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