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Remarks on stars and independent sets. (In English)

Aspects of topology, Mem. H. Dowker, Lond. Math. Soc. Lect. Note Ser. 93, 307-313 (1985).

[For the entire collection see Zbl 546.00024.]

A graph G has property I_k if every set of k independent vertices have a common neighbour; G has property I if it has property I_k for all k. Pach, proving a conjecture of Erdős and Fajtolowicz, has shown that theorem 1: Every triangle-free graph on n vertices and having property I, contains a vertex of valency at least $(n+1)/3$; if $3|(n+1)$, this result is best possible. Lemma: If every set of $\lceil \log n \rceil$ independent vertices of a triangle-free graph on n vertices have a common neighbour, then the graph has property I. Sharpening theorem 1, the authors prove theorem 4: Every triangle- free graph on n vertices and with property $I_{\lceil \log n \rceil}$ has a vertex of degree at least $(n+1)/3$.

Theorem 2: Let $f_r(n)$ denote the maximum integer such that every graph on n vertices having property I has a vertex of degree $\geq f$. Then $f_I(n) = (1+o(1))\frac{n \log\log n}{\log n}$.

Theorem 3. Let $f_I(r,n)$ denote the maximum integer f such that every K_r-free graph on n vertices having property I has a vertex of degree at least f. Then

$$f_I(w(n)\log n, n) \leq (2+o(1))\frac{n \log\log w(n)}{\log w(n)},$$

where $w(n)$ is an arbitrary function tending to infinity.

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