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Erdős, Paul; Sárközy, A.
On divisibility properties of integers of the form $a + a'$. (In English)
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Let $A \subseteq \{1, 2, ..., N\}$ be such that $a + a'$ for $a \in A$, $a' \in A$ be all square-free. Then the authors prove the following interesting results.
Theorem 1. For $N > N_0$ there exists an A such that $|A| > (1/248) \log N$.
Theorem 2. For $N > N_1$ every A satisfies $|A| < 3N^{3/4} \log N$. The proofs depend upon complicated applications of the large sieve and nice ingenuity which is characteristic of the authors. They remark that by slightly more complicated methods, they can get analogous results for k-free numbers. They remark that they can also consider the following problem. Let $A \subseteq \{1, 2, ..., N\}$ and $B \subseteq \{1, 2, ..., N\}$ such that $a + b$ is square-free for all $a \in A$ and $b \in B$. Then for $N \geq N_2$, $|A||B| < N^{3/2+\epsilon}$.
They can show that $|A||B|/N \to \infty$ is possible. They also remark that there is an absolute positive constant c such that $|A| > cN$, $|B| \to \infty$ is possible.

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Classification:
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arithmetic properties; sums of sequences of integers; square-free integers; k-free integers; large sieve