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Problems and results on additive properties of general sequences. III. (In English)

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Let $A = \{a_1 < a_2 < ...\}$ be an infinite sequence of positive integers, $A(N)$ be its counting function and $R(n)$ be the number of representations of n as a sum of two elements of A. In parts I and II [cf. Pac. J. Math. 118, 347-357 (1985; Zbl 569.10032), Acta Math. Hung. 48, 201-211 (1986; Zbl 621.10041)] the first two authors investigated how well $R(n)$ is approximable by "nice" functions. In this paper the boundedness of $|R(n+1) - R(n)|$ is studied. A simple example shows that the density $A(N)$ of A is not relevant in this question. Rather the number of blocks determines the answer. Let $B(N)$ be the number of blocks with starting elements up to N. It turns out that if $B(N)/N^{1/2}$ tends to infinity then $|R(n+1) - R(n)|$ can not be bounded, while there are examples of sets A with $B(N) \gg N^{-\epsilon}$ and $R(n)$ itself is bounded.
[Parts IV and V were published (with *V. Turán Sós*) in Lect. Notes Math. 1122, 85-104 (1985; Zbl 588.10056) and Monatsh. Math. 102, 183-197 (1986; Zbl 597.10055).]

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