
Zbl 704.11032
Erdős, Paul; Ivić, Aleksandar
The distribution of values of a certain class of arithmetic functions at consecutive integers. (In English)
Number theory. Vol. I. Elementary and analytic, Proc. Conf., Budapest/Hung. 1987, Colloq. Math. Soc. János Bolyai 51, 45-91 (1990).

[For the entire collection see Zbl 694.00005.]

The problem considered is that of the distribution of values of certain arithmetic functions, especially of the values at consecutive integers. The main motivation is the function $a(n)$, which counts the number of non- isomorphic Abelian groups of order n. One of the main results is

$$\sum_{n\leq x; a(n)=a(n+1)} 1 = Ax + O(x^{3/4}\log^4 x).$$

The result can be extended to nonnegative integer-valued arithmetic functions with squarefull kernel.
A great part of the paper deals with the functions $C(x)$ and $D(x)$, which denote the number of distinct values $a(n)$ for $n \leq x$ and the number of $n \leq x$ such that $n = a(m)$ for some m, respectively. For these functions lower bounds are given. The conjecture $C(x) = \exp(\log^{1/2+o(1)} x)$ and a similar one for $D(x)$ are proved if a certain conjecture involving the partition function is assumed.
Finally it is proved that there are infinitely many n such that the values $a(n+1), ..., a(n+t)$ are all distinct for $t = [C(\log n/\log\log n)^{1/2}]$ $(C > 0)$.

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Classification:
11N37 Asymptotic results on arithmetic functions
11N45 Asymptotic results on counting functions for other structures

Keywords:
asymptotic results; arithmetic functions; values at consecutive integers; number of non-isomorphic Abelian groups