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On some problems of the statistical theory of partitions. (In English)
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[For the entire collection see Zbl 694.00005.]
Let π be a generic "unrestricted" partition of the positive integer n, that is, a partition $\lambda_1 + \lambda_2 + ... + \lambda_m = n$, where the $\lambda_j' s$ are integers such that $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_m$, and let $p(n)$ be the number of such partitions. The number of conjugacy classes of the symmetric group of degree n is equal to $p(n)$, and the number of conjugacy classes of the alternating group of degree n is asymptotically equal to $p(n)/2$. By choosing a suitable prime summand, a proof that almost all partitions π of n have a summand which is > 1 and relatively prime to the other summands was given by *L. B. Beasley*, *J. L. Brenner*, *P. Erdős*, *M. Szalay* and *A. G. Williamson* [Period. Math. Hung. 18, 259-269 (1987; Zbl 617.20045)], and was used to simplify a proof originally given by Beasley, Brenner, and Williamson that almost all conjugacy classes of the alternating group of degree n contain a pair of generators.
It is now shown that the choice of a prime summand was necessary, in the sense that for almost all π's, if $\lambda_j > 1$ and $(\lambda_i, \lambda_j) = 1$ for each $i \neq j$ then λ_j is a prime. Also, let π^x be a generic unequal partition of n, that is, π^x represents a partition $\alpha_1 + \alpha_2 + ... + \alpha_m = n$, where the $\alpha_j' s$ are integers such that $\alpha_1 > \alpha_2 > ... > \alpha_m$, and let $M(\pi^x)$ denote the maximal number of consecutive summands in π^x. It is shown that for almost all π^x,

$$M(\pi^x) = (\log n)/(2\log 2) - (\log\log n)/(\log 2) + O(\omega(n)),$$

where $\omega(n) \to \infty$ (arbitrarily slowly). Finally, let $T_n(k)$ denote the number of solutions of $x^k = e$ in the symmetric group of degree n, where e is the identity element. Others have investigated the behavior of $T_n(k)$ as $n \to \infty$ for fixed $k \geq 2$. An estimate is now established for $T_n(k)$ for $1 \leq k \leq n^{(1/4)-\epsilon}$, $0 < \epsilon < 10^{-2}$, as $n \to \infty$.

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