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Bounds on the number of pairs of unjoined points in a partial plane. (In English)

Coding theory and design theory. Part II: Design theory, Proc. Workshop IMA Program Appl. Comb., Minneapolis/MN (USA) 1987-88, IMA Vol. Math. Appl. 21, 102-112 (1990).

[For the entire collection see Zbl 693.00019.]

A partial plane is a pair $\Sigma = (S, C)$ where S is a finite set (of points) and C is a collection of subsets of S (whose elements are called lines) such that no two points of Σ lie on more than one common line. The deficit D of a partial plane is the number of pairs of unjoined points in Σ. With $v = |S|$, $b = |C|$ and k denoting the cardinality of the largest line in Σ, the principal result is:

Theorem: Let n be an integer such that $n \geq 29$, $n^2 - n + 3 \leq v \leq n^2 + n - 1$, $b < v$, $k \leq v - 3$. Then $2D > 3v - 13v^{3/4}$.

This and other bounds given in the paper may be loosely phrased as saying that if a partial plane Σ has a small deficit then b is large relative to v. The proofs proceed by considering separately the cases when k is small or large relative to v. In the large line case linear programming techniques are utilized.

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Classification:

51E14 Finite partial geometries (general), nets, fractial spreads

05B30 Other designs, configurations

90C05 Linear programming

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linear space; projective plane; partial plane