Zbl 733.11003
Erdős, Paul; Kiss, Péter; Pomerance, Carl
On prime divisors of Mersenne numbers. (In English)
Acta Arith. 57, No.3, 267-281 (1991). [0065-1036]

Let f(n) be the sum of the reciprocals of the distinct prime divisors of the n-th Mersenne number $f(n)=\sum p^{-1}(p/2^n-1)$. By elementary, but complicated arguments the authors show that for each $k\geq 2$ and infinitely many n

$$\min(f(n),f(n+1),...,f(n+k-1))\geq \log_{k+2}n+c\log_{k+3}n$$

(c is an absolute negative constant, $\log_k n$ denotes the k-fold iterated logarithm). If the Extended Riemann Hypothesis for certain Dedekind zeta functions is assumed, then for all $k\geq 2$ und n sufficiently large the above min is $\leq 3\log_{k+2}n+ck$. Finally, the average order of f in short intervals is studied.

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Classification:
11A41 Elemementary prime number theory
11N37 Asymptotic results on arithmetic functions
11N25 Distribution of integers with specified multiplicative constraints

Keywords:
sum of reciprocals; distinct prime divisors; Mersenne number; average order; short intervals