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Erdős, Paul; Lacampagne, C.B.; Selfridge, J.L.

Estimates of the least prime factor of a binomial coefficient. (In English)

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We estimate the least prime factor p of the binomial coefficient $\binom{N}{k}$ for $k \geq 2$. The conjecture that $p \leq \max(N/k, 29)$ is supported by considerable numerical evidence. Call a binomial coefficient good if $p > k$. For $1 \leq i \leq k$ write $N - k + i = a_i b_i$, where b_i contains just those prime factors $> k$, and define the deficiency of a good binomial coefficient as the number of i for which $b_i = 1$. Let $g(k)$ be the least integer $N > k + 1$ such that $\binom{N}{k}$ is good. The bound $g(k) > ck^2/\ln k$ is proved. We conjecture that our list of 17 binomial coefficients with deficiency > 1 is complete, and it seems that the number with deficiency 1 is finite. All $\binom{N}{k}$ with positive deficiency and $k \leq 101$ are listed.

Classification:
11B65 Binomial coefficients, etc.
11N37 Asymptotic results on arithmetic functions

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least prime factor; binomial coefficient; deficiency of a good binomial coefficient; positive deficiency