Erdős Number Update

Jerrold W. Grossman
Department of Mathematics and Statistics
Oakland University, Rochester, MI 48309-4485
e-mail: `grossman@oakland.edu`

As a cultural reflection on the magnitude of Paul Erdős's contribution to mathematical research, the notion of *Erdős number* has been a part of the folklore of mathematicians for decades. It is defined inductively as follows. Paul has Erdős number 0. For each $n \geq 0$, a person not yet assigned an Erdős number who has a joint publication with a person having Erdős number n has Erdős number $n + 1$. Anyone who is not assigned an Erdős number by this process is said to have Erdős number ∞. Thus a person's Erdős number is just the distance from that person to Paul Erdős in the *collaboration graph C*, in which two authors are joined by an edge if they have a joint publication. For example, Albert Einstein has Erdős number 2, since he did not collaborate with Paul Erdős, but he did publish joint research with Ernst Straus, who was one of Paul's major collaborators. Purists can argue over how to count papers with more than two authors, but here we will adopt the liberal attitude that each of the $\binom{k}{2}$ pairs of authors in a k-author paper are adjacent in C.

In 1994 the author began a systematic process of data-collection for and study of Erdős numbers. In this brief article we want to provide the latest statistics. Because Paul Erdős died in 1996, the number of new coauthors is limited to those few people whose work was already in the pipeline; thus we do not expect the number of people with Erdős number 1 to rise much beyond its current value of 485. Hundreds of these coauthors are still very much alive and active, however, so the number of people with Erdős number 2, which currently stands at 5337, continues to grow rapidly, as does the order of the component of C containing Paul Erdős (which is clearly the largest component of C by far).

Annual updates of information related to Erdős numbers are given on the Web site of the Erdős Number Project (`http://www.oakland.edu/~grossman/erdoshp.html`). This site contains a list of all persons with Erdős number 1, together with their coauthors. The list is provided alphabetically, by year of first joint publication, and by number of joint papers. The coauthors of these people (i.e., persons with Erdős number 2) are also listed alphabetically, with their coauthors who have Erdős number 1. The site also has links to related information of all kinds, including numerous web sites and articles about Paul Erdős, as well as a dose of more whimsical items.

Figure 1 shows the distribution of the number of joint papers with Paul Erdős over the 485 persons who have written such papers. Not surprisingly, more than half of these people wrote only one joint paper with him, but 14 of them wrote more than 20 (these coauthors are listed in Table 1). Significant work arose in both camps, of course. For example, there

are only two joint papers with Alfred Tarski, but they spawned an industry in the study of large cardinals; and at the other end of the spectrum 57 papers over 30 years by Erdős and András Sárközy provided stunning results in the field of combinatorial number theory.

Figure 1. *The distribution of the number of joint papers with Paul Erdős (N = 485, median = 1, $\mu = 3.28$, $\sigma = 6.32$)*
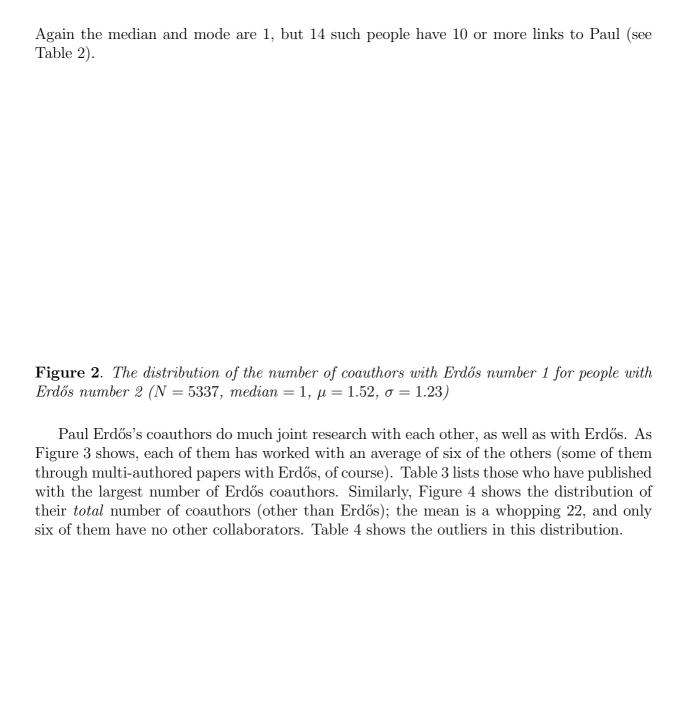
András Sárközy	57
András Hajnal	55
Ralph J. Faudree	46
Richard H. Schelp	40
Vera T. Sós	35
Cecil Clyde Rousseau	33
Alfréd A. Rényi	32
Endre Szemerédi	30
Pál Turán	30
Stefan Andrus Burr	27
Ronald L. Graham	27
Joel H. Spencer	23
Miklós Simonovits	22
Carl Pomerance	21

Table 1. *The people with the most papers with Erdős*

Linda M. Lesniak	18
Lowell W. Beineke	15
Dwight Duffus	13
Farrokh Saba	12
Pavel Pudlák	11
Walter Denis Wallis	11
Imre Bárány	10
Graham R. Brightwell	10
A. Robert Calderbank	10
Péter Hajnal	10
Gyula O. H. Katona	10
Michael D. Plummer	10
Paul D. Seymour	10
James B. Shearer	10

Table 2. *The people with Erdős number 2 having the most paths of length 2 to Erdős in C*

Many people who have not done research directly with Paul Erdős (or at least did not reach the point of publication) have worked heavily with his collaborators. Figure 2 shows the distribution of the number of Erdős coauthors over the 5337 persons with Erdős number 2.

Again the median and mode are 1, but 14 such people have 10 or more links to Paul (see Table 2).

Figure 2. *The distribution of the number of coauthors with Erdős number 1 for people with Erdős number 2 (N = 5337, median = 1, μ = 1.52, σ = 1.23)*

Paul Erdős's coauthors do much joint research with each other, as well as with Erdős. As Figure 3 shows, each of them has worked with an average of six of the others (some of them through multi-authored papers with Erdős, of course). Table 3 lists those who have published with the largest number of Erdős coauthors. Similarly, Figure 4 shows the distribution of their *total* number of coauthors (other than Erdős); the mean is a whopping 22, and only six of them have no other collaborators. Table 4 shows the outliers in this distribution.

Figure 3. *The distribution of the number of coauthors with Erdős number 1 for people with Erdős number 1 (N = 485, median = 3, μ = 5.69, σ = 6.83)*

Ronald L. Graham	42
Frank Harary	41
Vojtěch Rödl	38
Noga Alon	36
Zsolt Tuza	35
Joel H. Spencer	34

Table 3. *The people with the most coauthors with Erdős number 1*

Frank Harary	273
Noga Alon	161
Saharon Shelah	142
Ronald L. Graham	121
Charles J. Colbourn	120
Daniel J. Kleitman	115
Andrew M. Odlyzko	111

Table 4. *The people with Erdős number 1 having the most coauthors*

Figure 4. *The distribution of the total number of coauthors, other than Erdős, for people with Erdős number 1 (N = 485, median = 16, μ = 22.45, σ = 24.50)*

Readers with additions or corrections to any of the information in this article or on the Erdős Number Project web site are urged to communicate with the author.

June 9, 1998