ON THEOREMS OF CENTRAL FORCES

By

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On Theorems of Central Forces.

By Sir William R. Hamilton.

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Sir William R. Hamilton stated the following theorems of central forces, which he had proved by his calculus of quaternions, but which, as he remarked, might be also deduced from principles more elementary.

If a body be attracted to a fixed point, with a force which varies directly as the distance from that point, and inversely as the cube of the distance from a fixed plane, the body will describe a conic section, of which the plane intersects the fixed plane in a straight line, which is the polar of the fixed point with respect to the conic section.

And in like manner, if a material point be obliged to remain upon the surface of a given sphere, and be acted on by a force, of which the tangential component is constantly directed (along the surface) towards a fixed point or pole upon that surface, and varies directly as the sine of the arcual distance from that pole, and inversely as the cube of the sine of the arcual distance from a fixed great circle; then the material point will describe a spherical conic, with respect to which the fixed great circle will be the polar of the fixed point.

Thus, a spherical conic would be described by a heavy point upon a sphere, if the vertical accelerating force were to vary inversely as the cube of the perpendicular and linear distance from a fixed plane passing through the centre.

The first theorem had been suggested to Sir W. Hamilton by a recently resumed study of a part of Sir Isaac Newton's Principia; and he had been encouraged to seek for the second theorem, by a recollection of a result respecting motion in a spherical conic, which was stated some years ago to the Academy by the Rev. C. Graves. In that result of Mr. Graves, the fixed pole was a focus of the conic, and the polar was therefore the director arc; consequently, the sine of the distance from the polar was proportional to the sine of the distance from the pole, and, instead of the law now mentioned to the Academy, there was the simpler law of proportionality to the inverse square of the sine of the distance from the fixed pole or focus.