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On Cameron–Liebler line classes

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Abstract. Cameron–Liebler line classes are sets of lines in $PG(3,q)$ that contain a fixed number x of lines of every spread. Cameron and Liebler classified them for $x \in \{0,1,2,q^2-1,q^2,q^2+1\}$ and conjectured that no others exist. This conjecture was disproven by Drudge and his counterexample was generalised to a counterexample for any odd q by Bruen and Drudge. Nonexistence of Cameron–Liebler line classes was proven for different values of x by Penttila, Bruen and Drudge, Drudge, and Govaerts. In this paper, a new lower bound on x for existence of Cameron–Liebler line classes is obtained, and in the specific cases where q is a square or a cube, this new bound is improved upon.

1 Introduction

Cameron–Liebler line classes were introduced by Cameron and Liebler [9] in an attempt to classify collineation groups of $PG(n,q)$ that have equally many point orbits and line orbits. In their paper, they conjectured which groups these are. It is now known (T. Penttila, private communication, 2002) that the conjecture is true when the group is irreducible, but there is no classification yet of Cameron–Liebler line classes. In this paper, some new nonexistence results are presented.

Following Penttila [12], a *clique* in $PG(3,q)$ is either the set of all lines through a point P, denoted by $\mathsf{star}(P)$, or dually the set of all lines in a plane π, denoted by $\mathsf{line}(\pi)$. The planar pencil of lines in a plane π through a point P is denoted by $\mathsf{pen}(P,\pi)$.

There are many equivalent definitions for Cameron–Liebler line classes. Here three of them are listed: the first one because it is the most elegant one, the other ones because they will be useful later on.

Definition 1.1 (Cameron and Liebler [9], Penttila [12]). Let $\mathscr{L}$ be a set of lines in $PG(3,q)$ and let $\chi_{\mathscr{L}}$ be its characteristic function. Then $\mathscr{L}$ is called a *Cameron–Liebler line class* if one of the following equivalent conditions is satisfied.

1. There exists an integer x such that $|\mathscr{L} \cap \mathscr{S}| = x$ for all spreads $\mathscr{S}$.
2. There exists an integer x such that for every incident point-plane pair (P,π)

$$|\mathsf{star}(P) \cap \mathscr{L}| + |\mathsf{line}(\pi) \cap \mathscr{L}| = x + (q+1)|\mathsf{pen}(P,\pi) \cap \mathscr{L}|. \tag{1}$$

3. There exists an integer x such that for every line l of $\mathrm{PG}(3,q)$

$$|\{m \in \mathscr{L} : m \text{ meets } l, m \neq l\}| = (q+1)x + (q^2-1)\chi_{\mathscr{L}}(l). \tag{2}$$

It follows from the proof of the equivalence of these properties that the number x in each of these statements is the same. It is called the *parameter* of the Cameron–Liebler line class. We remark that the first definition implies that $x \in \{0, 1, 2, \dots, q^2+1\}$. Cameron and Liebler [9] showed that a Cameron–Liebler line class of parameter x consists of $x(q^2+q+1)$ lines and that the only Cameron–Liebler line classes for $x = 1$ are the cliques and for $x = 2$ the unions of two disjoint cliques. They also noticed that the complement of a Cameron–Liebler line class with parameter x is a Cameron–Liebler line class with parameter q^2+1-x. So, it suffices to study Cameron–Liebler line classes with parameter $x \leqslant \lfloor (q^2+1)/2 \rfloor$. Thus, the case $q = 2$ was immediately solved. In their paper, Cameron and Liebler conjectured that no other Cameron–Liebler line classes exist.

Penttila [12] shows that for $q \neq 2$ there exist no Cameron–Liebler line classes with parameter $x = 3$ or $x = 4$, with possible exception of the cases $(x, q) \in \{(4, 3), (4, 4)\}$. Bruen and Drudge [7] prove the nonexistence of Cameron–Liebler line classes with parameter $2 < x \leqslant \sqrt{q}$. Drudge [10] excludes the existence of a Cameron–Liebler line class with parameter $x = 4$ in $\mathrm{PG}(3,3)$, and proves that for $q \neq 2$ there exist no Cameron–Liebler line classes with parameter $2 < x \leqslant \varepsilon$, where $q+1+\varepsilon$ denotes the size of the smallest nontrivial blocking sets in $\mathrm{PG}(2,q)$, see Section 2. He also gives a counterexample to the conjecture of Cameron and Liebler: a Cameron–Liebler line class with parameter $x = 5$ in $\mathrm{PG}(3,3)$, in this way settling the case $q = 3$. Bruen and Drudge [8] then construct a Cameron–Liebler line class with parameter $x = (q^2+1)/2$ for any odd q. In Govaerts [11], the study of the case $x = 4$ is completed by showing that there exists no Cameron–Liebler line class with parameter $x = 4$ in $\mathrm{PG}(3,4)$.

In this paper, new bounds on x for nonexistence of Cameron–Liebler line classes are obtained. Theorem 4.5 gives a new bound for general $q \neq 2$, while Theorem 5.1 (Theorems 6.1 and 6.3) improves upon it for q square ($q = p^{3h}, p \geqslant 7$ prime, $h \geqslant 1$).

These theorems will be proved by studying how the lines of the Cameron–Liebler line class are distributed among the cliques of $\mathrm{PG}(3,q)$. In the proofs, these cliques will be assumed to be of the form $\mathsf{star}(P)$ for some point P, but the dual arguments show that the considered properties also hold for cliques of the form $\mathsf{line}(\pi)$ for a plane π.

To study the lines of the Cameron–Liebler line class in a clique, we follow Drudge's approach [10]. A clique $\mathscr{C}$ and its lines correspond to a projective plane and its points in the following way. If $\mathscr{C} = \mathsf{star}(P)$, then it suffices to take the quotient space with respect to P. If $\mathscr{C} = \mathsf{line}(\pi)$, then the dual plane can be considered. In this way, the lines of the line class in a clique correspond to a set of points in a plane.

2 Cameron–Liebler line classes and blocking sets

A *k-fold blocking set* in $\mathrm{PG}(2,q)$ is a set of points that intersects every line in at least k points. It is called *minimal* if no proper subset is a k-fold blocking set. A 1-fold blocking set is simply called a *blocking set*. It is called *trivial* if it contains a line. The following two lemmas show where (multiple) blocking sets show up in the study of Cameron–Liebler line classes.

Lemma 2.1 (Drudge [10]). *Let $\mathscr{L}$ be a Cameron–Liebler line class with parameter x. If $\mathscr{C}$ is a clique satisfying $x < |\mathscr{C} \cap \mathscr{L}| \leqslant q + x$, then $\mathscr{C} \cap \mathscr{L}$ forms a blocking set B in $\mathscr{C}$. If there exist no Cameron–Liebler line classes with parameter $x - 1$, then B is nontrivial.*

Lemma 2.2. *Let $\mathscr{L}$ be a Cameron–Liebler line class with parameter x. If $\mathscr{C}$ is a clique satisfying $x + \alpha(q+1) < |\mathscr{C} \cap \mathscr{L}|$, then $\mathscr{C} \cap \mathscr{L}$ forms an $(\alpha+1)$-fold blocking set in $\mathscr{C}$.*

Proof. Suppose that $\mathscr{C} = \mathsf{star}(P)$ is a clique satisfying $x + \alpha(q+1) < |\mathscr{C} \cap \mathscr{L}|$. Let π be any plane through P. By (1) and the fact that $|\mathsf{line}(\pi) \cap \mathscr{L}|$ is at least zero, it can be concluded that $|\mathsf{pen}(P, \pi) \cap \mathscr{L}|$ is greater than α. □

Blocking sets are much-studied objects. Below, some results are listed that will be used later on. In these theorems, c_p equals $2^{-1/3}$ when $p \in \{2, 3\}$ and 1 when $p \geqslant 5$.

Theorem 2.3. *Let B be a nontrivial blocking set of* $\mathrm{PG}(2, q)$, $q > 2$.

1. (Blokhuis [2]) *If q is a prime, then $|B| \geqslant 3(q+1)/2$.*
2. (Bruen [5]) *If q is a square, then $|B| \geqslant q + \sqrt{q} + 1$.*
3. (Blokhuis [3], Blokhuis et al. [4]) *If $q = p^{2e+1}$, p prime, $e \geqslant 1$, then $|B| \geqslant \max(q + 1 + p^{e+1}, q + 1 + c_p q^{2/3})$.*

Theorem 2.4. *Let B be a blocking set in* $\mathrm{PG}(2, q)$, *q square, containing neither a line nor a Baer subplane.*

1. (Blokhuis et al. [4]) *If $q > 16$, $q = p^h$, p prime, then $|B| \geqslant q + 1 + c_p q^{2/3}$.*
2. (Szőnyi [16]) *If $q = p^2$, p prime, then $|B| \geqslant 3(q+1)/2$.*

Theorem 2.5 (Polverino and Storme [15]). *In* $\mathrm{PG}(2, q)$, $q = p^{3h}$, *$p \geqslant 7$ prime, $h \geqslant 1$, the smallest minimal nontrivial blocking sets that are not Baer subplanes are*:

1. *a minimal blocking set of size $q + p^{2h} + 1$, projectively equivalent to the set $K = \{(x, T(x), 1) : x \in \mathrm{GF}(q)\} \cup \{(x, T(x), 0) : x \in \mathrm{GF}(q) \backslash \{0\}\}$, with T the trace function from* $\mathrm{GF}(q)$ *to* $\mathrm{GF}(p^h)$;
2. *a minimal blocking set of size $q + p^{2h} + p^h + 1$, projectively equivalent to the set $K = \{(x, x^{p^h}, 1) : x \in \mathrm{GF}(q)\} \cup \{(x, x^{p^h}, 0) : x \in \mathrm{GF}(q) \backslash \{0\}\}$.*

(Polverino [14]) *If $h = 1$, then these are the only minimal nontrivial blocking sets of size smaller than* $3(q+1)/2$.

Lines of $\mathrm{PG}(2, q)$ intersect these blocking sets in the following way. Denote p^h by q_0. Apart from tangent lines, the first one has $q_0 + 1$ $(q_0^2 + 1)$-secants and q_0^4 $(q_0 + 1)$-secants; the second one has one $(q_0^2 + q_0 + 1)$-secant and $q_0^4 + q_0^3 + q_0^2$ $(q_0 + 1)$-secants.

Theorem 2.6. *Let B be a k-fold blocking set of* $\mathrm{PG}(2,q)$, $k > 1$.

1. (Bruen [6]) *If B contains a line, then* $|B| \geqslant kq + q - k + 2$.
2. (Ball [1]) *If B does not contain a line, then* $|B| \geqslant kq + \sqrt{kq} + 1$.

3 Two lemmas

Lemma 3.1. *Let $\mathscr{L}$ be a Cameron–Liebler line class with parameter $x < q^2 + 1$. Then there exists a clique containing at most x lines of $\mathscr{L}$.*

Proof. Let l be a line not in $\mathscr{L}$. By (2), there are $(q+1)x$ lines of $\mathscr{L}$ meeting l. This implies that there exists a point P on l that satisfies $|\mathsf{star}(P) \cap \mathscr{L}| \leqslant x$. □

Lemma 3.2. *If $\mathscr{L}$ is a Cameron–Liebler line class with parameter* $0 < x \leqslant q$, *then there exists a clique $\mathscr{C}$ satisfying $x < |\mathscr{C} \cap \mathscr{L}| \leqslant q + x$.*

Proof. Suppose that $\mathscr{L}$ is a Cameron–Liebler line class with parameter $0 < x \leqslant q$ and that there exists no clique $\mathscr{C}$ satisfying $x < |\mathscr{C} \cap \mathscr{L}| \leqslant q + x$.

Suppose that $\mathscr{C} = \mathsf{star}(P)$ is a clique satisfying $0 < |\mathscr{C} \cap \mathscr{L}| \leqslant x$. Then there exists a plane π through P containing exactly one line of $\mathscr{C} \cap \mathscr{L}$. By (1), $q + 1 \leqslant |\mathsf{line}(\pi) \cap \mathscr{L}| < q + x$, a contradiction. Dually, there exists no plane π satisfying $0 < |\mathsf{line}(\pi) \cap \mathscr{L}| \leqslant x$.

Suppose that $\mathscr{C} = \mathsf{star}(P)$ is a clique satisfying $|\mathscr{C} \cap \mathscr{L}| = 0$. Then every plane π through P satisfies $|\mathsf{pen}(P, \pi) \cap \mathscr{L}| = 0$. By (1), $|\mathsf{line}(\pi) \cap \mathscr{L}| = x$, a contradiction with the preceding paragraph.

The previous observations show that there exist no cliques containing at most x lines of $\mathscr{L}$, a contradiction by Lemma 3.1. □

4 The general case

In this section, assume that $\mathscr{L}$ is a Cameron–Liebler line class in $\mathrm{PG}(3,q)$, $q > 2$, with parameter $x \leqslant q$, and that no Cameron–Liebler line classes with parameter $x - 1$ exist. Recall that Penttila [12] proves that for $q > 2$, no Cameron–Liebler line classes with parameter 3 exist. Let $q + 1 + \varepsilon$ denote the size of the smallest nontrivial blocking sets in $\mathrm{PG}(2,q)$.

Lemma 4.1. *There exists no clique $\mathscr{C}$ satisfying* $x < |\mathscr{C} \cap \mathscr{L}| \leqslant q + \min(x, \varepsilon)$.

Proof. Immediate from Lemma 2.1 and the definition of ε. □

Corollary 4.2 (see also Drudge [10]). *There exist no Cameron–Liebler line classes with parameter* $2 < x \leqslant \varepsilon$.

Proof. In this case Lemma 4.1 contradicts Lemma 3.2. □

For the rest of this section, assume that $x > \varepsilon$.

Lemma 4.3. *There exists no clique $\mathscr{C}$ satisfying* $x - \varepsilon < |\mathscr{C} \cap \mathscr{L}| < q + 1$.

Proof. If $\mathscr{C} = \mathsf{star}(P)$ were a clique satisfying $x - \varepsilon < |\mathscr{C} \cap \mathscr{L}| < q + 1$, then there would exist a plane π through P for which $|\mathsf{pen}(P, \pi) \cap \mathscr{L}| = 1$. By (1), this plane satisfies $x < |\mathsf{line}(\pi) \cap \mathscr{L}| \leqslant q + \varepsilon$, a contradiction with Lemma 4.1. □

Lemma 4.4. *There exists no clique $\mathscr{C}$ satisfying* $0 \leqslant |\mathscr{C} \cap \mathscr{L}| < \varepsilon$.

Proof. If $\mathscr{C} = \mathsf{star}(P)$ were a clique satisfying $0 \leqslant |\mathscr{C} \cap \mathscr{L}| < \varepsilon$, then there would exist a plane π through P for which $|\mathsf{pen}(P, \pi) \cap \mathscr{L}| = 0$. By (1), this plane satisfies $x - \varepsilon < |\mathsf{line}(\pi) \cap \mathscr{L}| \leqslant x$, a contradiction with Lemma 4.3. □

Theorem 4.5. *In* $\mathrm{PG}(3, q)$, $q > 2$, *there exist no Cameron–Liebler line classes with parameter* $2 < x < 2\varepsilon$, *where* $q + 1 + \varepsilon$ *denotes the size of the smallest nontrivial blocking sets in* $\mathrm{PG}(2, q)$.

Proof. If $x < 2\varepsilon$, then the intervals of Lemmas 4.3 and 4.4 partially overlap, implying that there exists no clique containing less than $q + 1$ lines of $\mathscr{L}$. This is contradictory to Lemma 3.1. □

Corollary 4.6. *In* $\mathrm{PG}(3, q)$, *q prime*, $q > 2$, *there exist no Cameron–Liebler line classes with parameter* $2 < x \leqslant q$.

Proof. Use Theorem 2.3, Part 1. □

5 Improvements for *q* square

Theorem 5.1. *In* $\mathrm{PG}(3, q)$, *q square, there exist no Cameron–Liebler line classes with parameter* $2 < x \leqslant \min(\varepsilon', q^{3/4})$, *where* $q + 1 + \varepsilon'$ *denotes the size of the smallest nontrivial blocking sets in* $\mathrm{PG}(2, q)$ *not containing a Baer subplane.*

Proof. Suppose that $\mathscr{L}$ is a Cameron–Liebler line class with parameter $2 < x \leqslant \min(\varepsilon', q^{3/4})$, and assume that no Cameron–Liebler line classes with parameter $x - 1$ exist.

Suppose that $\mathscr{C} = \mathsf{star}(P)$ is a clique satisfying $x < |\mathscr{C} \cap \mathscr{L}| \leqslant q + x$. By Lemma 2.1 and the restriction $x \leqslant \varepsilon'$, in the plane corresponding to $\mathscr{C}$, $\mathscr{C} \cap \mathscr{L}$ contains a Baer subplane B. Since there are at most $x - \sqrt{q} - 1$ points of $\mathscr{C} \cap \mathscr{L}$ outside B, there exists a $(\sqrt{q} + 1)$-secant to $\mathscr{C} \cap \mathscr{L}$. Denote the corresponding plane through P by π. Since $|\mathsf{pen}(P, \pi) \cap \mathscr{L}| = \sqrt{q} + 1$, it follows from (1) that $q\sqrt{q} + \sqrt{q} + 1 \leqslant |\mathsf{line}(\pi) \cap \mathscr{L}| \leqslant q\sqrt{q} + x$. By Lemma 2.2, $\mathsf{line}(\pi) \cap \mathscr{L}$ is a $\sqrt{q}$-fold blocking set in $\mathsf{line}(\pi)$. Comparing the upper bound on $|\mathsf{line}(\pi) \cap \mathscr{L}|$ with the known lower bounds for the size of multiple blocking sets from Theorem 2.6 yields a contradiction.

So, in contradiction with Lemma 3.2, there exists no clique $\mathscr{C}$ satisfying $x < |\mathscr{C} \cap \mathscr{L}| \leqslant q + x$. □

Corollary 5.2. *Let q be a square, $q = p^h$, p prime.*

1. *If $q > 16$ then there exist no Cameron–Liebler line classes in* $\mathrm{PG}(3,q)$ *with parameter $2 < x \leqslant c_p q^{2/3}$, where c_p equals $2^{-1/3}$ when $p \in \{2,3\}$ and 1 when $p \geqslant 5$.*
2. *If $p > 3$ and $h = 2$, then there exist no Cameron–Liebler line classes in* $\mathrm{PG}(3,q)$ *with parameter $2 < x \leqslant q^{3/4}$.*

Proof. Immediate by Theorems 5.1 and 2.4. □

6 Improvements for $q = p^{3h}$

Theorem 6.1. *Let $q = p^{3h}$, $p \geqslant 7$ prime, $h \geqslant 1$ odd, and let $q + 1 + \varepsilon''$ denote the size of the smallest nontrivial blocking sets in* $\mathrm{PG}(2,q)$ *containing neither a minimal blocking set of size $q + p^{2h} + 1$, nor one of size $q + p^{2h} + p^h + 1$. In* $\mathrm{PG}(3,q)$ *there exist no Cameron–Liebler line classes with parameter $2 < x \leqslant \min(\varepsilon'', q^{5/6})$.*

Proof. Suppose that $\mathscr{L}$ is a Cameron–Liebler line class with parameter $2 < x \leqslant \min(\varepsilon'', q^{5/6})$, and assume that no Cameron–Liebler line classes with parameter $x - 1$ exist.

Suppose that $\mathscr{C} = \mathsf{star}(P)$ is a clique satisfying $x < |\mathscr{C} \cap \mathscr{L}| \leqslant q + x$. By Lemma 2.1 and the restriction $x \leqslant \varepsilon''$, in the plane corresponding to $\mathscr{C}$, $\mathscr{C} \cap \mathscr{L}$ contains either a minimal blocking set of size $q + p^{2h} + 1$ or one of size $q + p^{2h} + p^h + 1$. In both cases, $\mathscr{C} \cap \mathscr{L}$ has a $(p^{2h} + 1 + a)$-secant for some $0 \leqslant a \leqslant x - p^{2h} - 1$. Let π be the plane through P defined by this secant. By (1), it satisfies $(q+1)(p^{2h} + a) + 1 \leqslant |\mathsf{line}(\pi) \cap \mathscr{L}| < x + p^{2h}q + aq + a + 1$. By Lemma 2.2, $\mathsf{line}(\pi) \cap \mathscr{L}$ forms a $(p^{2h} + a)$-fold blocking set in $\mathsf{line}(\pi)$. However, comparing the upper bound for $|\mathsf{line}(\pi) \cap \mathscr{L}|$ with the known lower bounds for the size of multiple blocking sets from Theorem 2.6 yields a contradiction.

So, in contradiction with Lemma 3.2, there exists no clique $\mathscr{C}$ satisfying $x < |\mathscr{C} \cap \mathscr{L}| \leqslant q + x$. □

Corollary 6.2. *Let $q = p^3$, $p \geqslant 7$ prime. There exist no Cameron–Liebler line classes in* $\mathrm{PG}(3,p^3)$ *with parameter $2 < x \leqslant q^{5/6}$.*

Proof. In this case $\varepsilon'' = (q+1)/2$, see Theorem 2.5. □

Theorem 6.3. *Let $q = p^{3h}$, $p \geqslant 7$ prime, $h > 1$ even, and let $q + 1 + \varepsilon''$ denote the size of the smallest nontrivial blocking sets in* $\mathrm{PG}(2,q)$ *containing neither a Baer subplane, nor a minimal blocking set of size $q + p^{2h} + 1$, nor one of size $q + p^{2h} + p^h + 1$. In* $\mathrm{PG}(3,q)$ *there exist no Cameron–Liebler line classes with parameter $2 < x \leqslant \min(\varepsilon'', q^{3/4})$.*

Proof. A combination of the proofs of Theorems 5.1 and 6.1 yields this result. □

For the proof of the following corollary, a few more definitions are needed. A blocking set B in $\mathrm{PG}(2,q)$ is called *small* when it consists of less than $3(q+1)/2$ points. If $q = p^h$, p prime, then the maximal integer e for which every line intersects B in 1 (mod p^e) points is called the *exponent* of B. It follows from results of Szőnyi [16] that a small minimal nontrivial blocking set B in $\mathrm{PG}(2,q)$, $q = p^h$, p prime, $p \geqslant 7$, has exponent $1 \leqslant e \leqslant h/2$, and that the size of B must lie in certain intervals depending on e.

Corollary 6.4. *Let $q = p^6$, $p \geqslant 7$ prime. There exist no Cameron–Liebler line classes in* $\mathrm{PG}(3,q)$ *with parameter $2 < x \leqslant q^{3/4}$.*

Proof. By Theorem 6.3, it suffices to show that $\varepsilon'' \geqslant q^{3/4}$. Suppose that this is not the case, i.e., suppose that there exists a minimal nontrivial blocking set different from the three enumerated in Theorem 6.3 of size smaller than $q+1+q^{3/4}$. A result of Polverino and Storme [15] says that the exponent e of this blocking set must be 1. But a small minimal blocking set with exponent $e = 1$ has size at least $q+1+p(q/p+1)/(p+1)$, see Polverino [13] and Szőnyi [16]. This number is larger than $q+1+q^{3/4}$, a contradiction. $\square$

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