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THEORY OF THE ZERO ORDER EFFECT SUITABLE TO INVESTIGATE THE SPACE-TIME GEOMETRICAL PROPERTIES

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Abstract. The applicability of Einsteins relativity theory on galactic scale and the role of geometry for the solution of the problems of observational astrophysics are discussed. The theory of the zero order effect to study the geometrical properties of space-time in experiment is given.

1. Introduction

The declinations of the planets orbits from ideal circles experimentally discovered by I. Kepler in XVII century posed a dilemma. Either the laws of Nature and Mathematics were not identical, and the mathematical harmony did not rule the Universe, or our knowledge was not complete not only in Science but in Mathematics as well.

In the end of XVIII century C. Gauss became the first who approached the problem of the applicability of the Euclidean geometry to the World in a constructive way. He measured the sum of the interior angles of the triangle in situ directly. The vortices of the triangle were at the peaks of the surrounding mountains. Gauss did not find any deviation in the geometry of the world and Euclidean geometry within the accuracy of his measurements.

In the beginning of the XIX century N. Lobachevsky considered and evaluated the principal possibilities of the astronomical measurements, and this inevitably lead him to the construction of the first non-Euclidean geometry.

In the middle of the XIX c. W. Clifford proclaimed and successively defended his idea that no physical phenomenon can be experimentally distinguished from the corresponding change of the geometrical curvature of the World.

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Following these ideas A. Einstein in the beginning of XX c. reduced the general but qualitative Clifford's statement to the more narrow but quantitative theory. He demanded the general covariance of physical laws, postulated the invariance of the light speed and the equivalence principle and produced a theory according to which the unavoidable gravitation could not be distinguished from the geometrical properties of space-time. In his fundamental paper Einstein considered the space-time described by Riemann-Minkowski geometry, predicted the effects that could be experimentally observed in this case and gave the calculation of them. The experiments revealed the good correlation with the calculation, and geometry became the full right part of physical theory. On the macro level, it made the Newton theory of Solar system gravity more precise. On the micro level, the Dirac theory was introduced into quantum mechanics. On mega level, the cosmology obtained the expanding Universe theory and the accompanying circumstances.

When we discuss the experimental data that have to be compared with the theory, we should mention the scale. There are three such scales in astrophysics: Solar system, galaxy and all the observable Universe. The known achievements of the general relativity theory (GRT) based on the introduction of the new (Riemann) geometry provided the link between the laboratory physics and the first of these scales. In the end of the XX c. there appeared the astrophysical data that can not be explained by the theory without new notions like dark matter or dark energy or without the modification of the foundations of the theory including the geometry of space-time. When choosing the new geometry one should start with the analysis of the problems appearing already on the galactic scale.

Suggesting the physical effect demonstrating the geometrical properties of space-time, one should pay attention to the fact that the static spherically symmetric solutions of the gravity equations both for Riemann geometry and for its generalization, Finsler geometry, give the same observable predictions. The effect that could be used for such investigation is the gravitational radiation, i.e. gravitational waves (GW). The existence of the GW predicted by the GRT has an indirect experimental support: the change of the orbital period in some double star systems [18]. For the different geometries of space-time the GW would possess different properties. But though there are several physical effects that could be used to investigate these properties, the problem of the direct observation of the GW is not solved up to now. This is due to the utmost smallness of the perturbation that the GW produce on any of the known physical effects already in the first order of magnitude. Since the GW are waves, we can use the resonance phenomena that could appear not in the first order of the perturbation theory as it was suggested in various approaches up to now but in the zero order.

The material is organized as follows. We consider the metrical approach to the gravitation theory to be valid. Since the geometry appears to be closely connected to the mass distribution, let us first give some results of the recent astrophysical observations and discuss their possible interpretations. Then we will point out some additional details concerning the space-time geometry apart from those that follow from the experimental data. Then the theory of the optic-metrical parametric resonance (OMPR) will be discussed and its results and interpretations for various cases will be analyzed. Finally, the examples of the astrophysical systems suitable for the observations are given.

2. Experimental data and its interpretation

The results of the astrophysical observations are the following. On the galactic scale, the rotation curves, i.e. the dependencies of the stars orbital velocities on their distances to the galaxies centers were measured for some galaxies [7], [13], [33]. On the Universe scale, the GRT effect of the gravitational lensing on the galactic clusters is found. This supports the Einstein idea about the link between the metric and gravity, but the result is several times larger than the GRT prediction. The acceleration of the Universe expansion is ascertained [31]-[32], and this leads to the notion of the dark energy.

The review of the theoretical results is given in [23]. According to the Introduction let us give a brief list of the results and ideas concerning only the galactic scale phenomena. To illustrate them let us give a figure from [8], Fig.1.

The experimental points obtained when measuring the orbital velocities, v, of stars of the spiral galaxies vis. their distances to the centers of those galaxies, R, can be described by the empirical formula [22]

$$v^2 = \frac{\beta^* c^2 N^*}{R} + \frac{\gamma^* c^2 N^* R}{2} + \frac{\gamma_0 c^2 R}{2} \tag{1}$$

where c is the light speed, N^* is the number of stars in the galaxy (usually about 10^{11}), β^* for the Sun is $\beta^* = \frac{M_S G}{c^2}$cm ($M_S$ is the Solar mass, G is the gravity constant), γ^* and γ_0 are universal parameters $\gamma^* = 5.42 \cdot 10^{-41}\text{cm}^{-1}$, $\gamma_0 = 3.06 \cdot 10^{-30}\text{cm}^{-1}$. All the three parameters become of the same order at the border of a galaxy, while the result of the Newton theory as well as the Schwarzschilds solution of the GRT equations predict only the decrease of the velocity corresponding to the first term in Eq.(1). The calculations were performed with regard to the exponential distribution of stars in a galaxy. To provide the observed motion of the gleaming stars, the existence of additional matter interacting with the stars gravitationally is suggested. The mass of this matter must be thrice as much as the mass of the visible stars, it must be located at the periphery of a galaxy and it neither emits, nor absorbs the electromagnetic radiation. In this paper it is essential to underline that the same effects take place for the clusters of galaxies too [23], that is on a Universe scale. That is

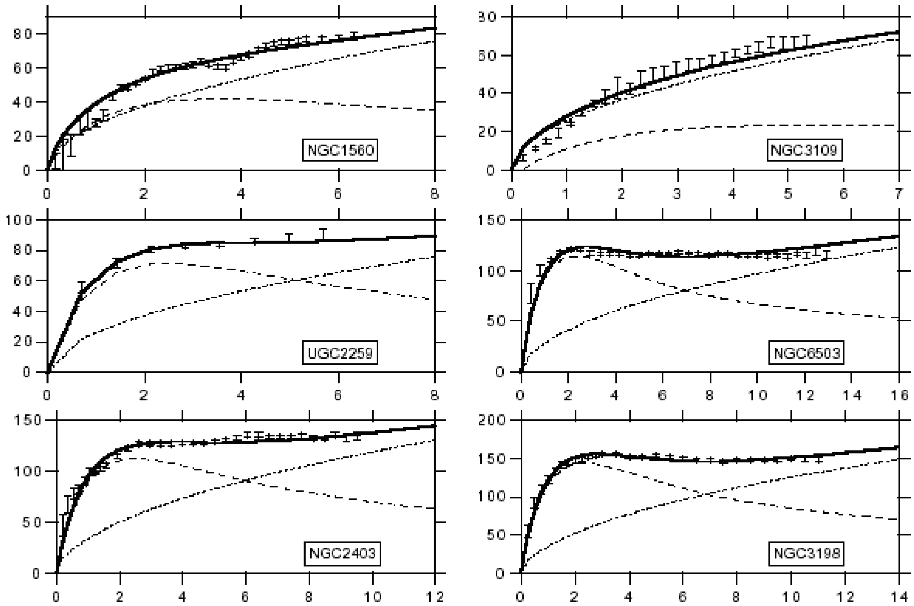

FIGURE 1. Orbital velocities (km/s) as functions of R/R_0, where R_0 is a characteristic scale for each galaxy. Dashed line is the Newtonian potential (coinciding with the Schwarzschilds solution), produced by the observable gleaming matter with regard to the exponential distribution of stars inside the galaxy [7].

why it is desirable to have the same explanation for both scales and not involve additional reasons.

The efforts of the theoreticians aimed at the solution of the problem have two directions. The first is the construction of a theory of the hypothetical elementary particles forming the dark matter. The second suggests modifying the existing theory of space-time and gravitation in such a way that there is no need for the extra type of matter. For any change of the theory the natural test is the preservation of the existing phenomenology, particularly, Newton gravity law for the Solar system scale and two other GRT effects following from the Schwarzschilds solution.

Let us now briefly mention some approaches belonging to the second direction.

I. The most straightforward approach is the successive complication of the quadratic expression for the Einstein-Hilbert action

$$S_{EH} = -\frac{c^3}{16\pi G}\int d^4x(-g)^{1/2}R^{\alpha}_{\alpha} \tag{2}$$

with account to the metric terms of the higher orders. For example [15],

$$S_{W_1} = -\frac{c^3}{16\pi G}\int d^4x(-g)^{1/2}(R^\alpha_\alpha)^2 \tag{3}$$

or

$$S_{W_2} = -\frac{c^3}{16\pi G}\int d^4x(-g)^{1/2}R^{\alpha\beta}_{\alpha\beta} \tag{4}$$

The corrections due to Eqs.(3) or (4) must give a negligibly small contribution to the Schwarzschilds solution. Besides, already this approach makes it possible to regard the cosmological constant in a way Einstein tried to do it himself.

II. Another natural approach is the introduction of an additional macroscopic gravitational field, S, usually the scalar one. For example [14]

$$S_{BD} = -\frac{c^3}{16\pi G}\int d^4x(-g)^{1/2}(SR^\alpha_\alpha - w\frac{S_{;\mu}S^{;\mu}}{S}) \tag{5}$$

where w is a constant.

III. The third approach is the increase of the number of the space-time dimensions with the subsequent transfer to the Plancks scale of lengths. The corresponding works began from [19] and then lead to the mathematically developed modern theories of strings [25] and then of branes [30].

Let us now mention the approaches providing not only the specification of the already existing structures in order to get the solution that is closer to the observations, but the approaches aimed at the revision of the structures themselves presumably giving the same result.

IV. The classical foundation can be also revised. The MOND phenomenological approach (MOdified Newton Dynamics) was suggested in [26] to introduce the new world constant with the dimension of an acceleration

$$\mu(\frac{a}{a_0})\overrightarrow{a} = \overrightarrow{f} \text{ or } \overrightarrow{a} = \nu(\frac{f}{a_0})\overrightarrow{f} \tag{6}$$

It was suggested to find such functions $\mu(x)$ or $\nu(x)$ and such value of a_0 that they match the classical result for the Solar system scale and give Eq.(1) for the galaxy scale. The relativistic generalization of MOND was performed in [9] where the scalar field ψ was introduced to give an additional term to the expression of Einstein-Hilbert action in the form

$$S(\psi) = -\frac{1}{8\pi GL^2}\int d^4x(-g)^{1/2}f(L^2g^{\alpha\beta}\psi_{;\alpha}\psi_{;\beta}) \tag{7}$$

(f is a scalar function, L is constant). After that the MOND theory can not be regarded as a pure phenomenology. Naturally, this approach gives a good fit for the observed rotation curves described by Eq.(1).

In fact, it does not matter if we speak about the dark matter or a scalar field in the gravitation theory, or about the ether in electrodynamics - in both cases the object of discussion acts on observable bodies but can not be detected itself.

But the same can be said about the geometry of the world. The principal idea of relativity stemming from Lobachevskys work and formulated by Einstein is that one should not oppose gravitation and geometry but regard them in the non-separable connection.

V. The geometry of space-time can be also modified. The rejection of symmetry in metrics indices [21],[27] can also lead to the suitable description of the rotation curves while dark matter is not needed.

VI. Already in 1918 H. Weyl [42] stepped aside from the Riemannian geometry suggested by Einstein in order to unify gravitation and electromagnetism with the help of metrics. He suggested the transformations of the following form

$$g_{\mu\nu}(x) \to e^{2\alpha(x)} g_{\mu\nu}(x) \tag{8}$$

$$A_\mu(x) \to A_\mu(x) - e\partial_\mu \alpha(x) \tag{9}$$

Here the gravitation and electromagnetism are united by the common function $\alpha(x)$, and this leads to the new Weylian geometry. The equations that can be obtained in this approach do not give the regular Einstein equations; nevertheless, they contain the Schwarzschilds solution for the Solar system scale. Weyl called Eq.(8) the gauge transformation, i.e. dependent on scale, but later this term was adopted by the other fields of physics mainly for the cases when the exponent was imaginary. In gravitation theory such transformations are now called conformal.

VII. The further evolution of these ideas leads to the theories of conformal gravitation where the metrics has an additional symmetry, corresponding to Eq.(8), the electromagnetic variables are not involved and this means that the geometry remains Riemannian. Formally such approach is analogous to I, but the choice of coefficients in Eqs. (3) and (4) is specific. The Einstein equations that appear in this approach are [24]

$$4\alpha_g W^{\mu\nu} = 4\alpha_g (2C^{\mu\lambda\nu\kappa}_{;\lambda;\kappa} - C^{\mu\lambda\nu\kappa} R_{\lambda\kappa}) = T^{\mu\nu} \tag{10}$$

where α_g is a dimensionless constant, $C^{\mu\lambda\nu\kappa}$ is the so called Weyl tensor which doesnt change with transformations Eq.(8). Then in [24] they change

$$\begin{aligned} W^{\mu\nu}(x) &\to e^{-6\alpha(x)} W^{\mu\nu}(x) \\ T^{\mu\nu}(x) &\to e^{-6\alpha(x)} T^{\mu\nu}(x) \end{aligned}$$

transform the coordinates with the use of a certain function $B(r)$ and introduce the source function $f(r)$. As a result the stationary version of Eq.(10) gives the Poisson equation but not of the second order as usual, but of the fourth order

$$\nabla^4 B(r) = f(r) \tag{11}$$

If there is a spherical symmetry, the Eq.(11) has an exact solution. And this solution not only contains the term corresponding to the Newton-Schwarzschilds solution but also the terms corresponding to Eq.(1)

$$B(r > R) = -g_{00} = 1 - \frac{2\beta}{r} + \gamma r \tag{12}$$

$$2\beta = \frac{1}{6}\int_0^R dr' r'^4 f(r'); \gamma = \frac{1}{2}\int_0^R dr' r'^2 f(r') \tag{13}$$

Solid lines on Fig.1 correspond to the results of the conformal gravitation approach to the galactic rotation curves. The fits are good. The described approach does not need the introduction of the additional (dark) matter, i.e. the additional scalar field. Instead it uses another choice of the scalar function when formulation of the variation principle. This preserves the Riemannian geometry of space-time but leads to the Einstein equation of the form of Eq.(10) which by the way does not have the structure of the wave equation for the empty space. This means that the GW do not exist, and the effect described in [18] which coincide with the prediction of the traditional GRT within very high accuracy must be explained in some other way.

The material discussed in this Section suggests the following conclusion. The successful modifications of the theory that correlate with the experimental data point at the possible existence of the additional terms in the gravitation law, their role depending on the chosen scale. Preserving Riemannian geometry one has to chose one of the following:

- either to search for an additional dark matter, located at the periphery of a galaxy;
- or to describe the gravitation on the scale of a galaxy using another scalar when formulating the variation principle for the action.

3. Finslerian geometry of the anisotropic space-time

Apart from the scale, one has to pay attention to another important thing. The data present on Fig.1 and those analogous to them do mainly concern the spiral galaxies that have expressed (space) anisotropy. But the notion of an isotropy could be regarded in a broader sense. The generalization of the GRT for the anisotropic space-time in which, for example, the light speed varies and depends on the direction, was performed in [29] where the theory is based on Finsler geometry. The metrics in Finsler geometry depends not only on the coordinate of a point (x^α) as in Riemannian geometry, but on a certain tangent vector too, ($\dot{x}^\alpha$) = $\frac{dx^\alpha}{dt}$ (t is a parameter). Usually, [35], this metrics is presented as

$$g_{\mu\nu}(x, \dot{x}) = \frac{1}{2}\frac{\partial^2 F^2(x, \dot{x})}{\partial \dot{x}^\mu \, \partial \dot{x}^\nu} \tag{14}$$

where $F(x,\dot{x})$ is a smooth, scalar, homogeneous of the first order, positive function with determinant $\det|\frac{\partial^2 F^2(x,\dot{x})}{\partial \dot{x}^\mu \partial \dot{x}^\nu}| \neq 0$. One of the principal results obtained in [29] is the proof that the analogues of Einstein equation in Finsler case (for various metrics) have Schwarzschilds solutions. It is also shown that within the same accuracy of measurements performed in the Solar system, it is impossible to distinguish these solutions from those of the GRT. Two other effects (the light bending when passing close to the Sun and the red shift) are present both in Riemann and Finsler cases though for different reasons. That is why these effects cant be used to make a justified choice of geometry to describe the real space-time.

Finsler geometry can be involved into the traditional approach by the use of a special metrics in tangent space. This metrics consists of two parts one of which can depend not only on the coordinate but on the vector direction as well. If one performs a conformal transformation with the 'horizontal' part, the corresponding corrections of the 'vertical' part would affect the Einstein equations. In this case they present a system of equations for the corresponding tensors [6].

There are several additional reasons to turn to a special case of Finsler geometry - to the spaces with the Berwald-Moor metrics which corresponds to

$$F(y) = (y^1 y^2 y^3 y^4)^{1/4}; y^\alpha = \frac{dx^\alpha}{dt} \tag{15}$$

In [10] it is shown that the well-known (physical) problem of the spontaneous symmetry break in the fermion-antifermion condensate corresponds to the (geometrical) partial or complete isotropy break of the space-time if its metrics can be described as

(16)
$$\begin{aligned} ds' = {} & (dx_0 - dx_1 - dx_2 - dx_3)^{\frac{1+r_1+r_2+r_3}{4}} (dx_0 - dx_1 + dx_2 + dx_3)^{\frac{1+r_1-r_2-r_3}{4}} \\ & \cdot (dx_0 + dx_1 - dx_2 + dx_3)^{\frac{1-r_1+r_2-r_3}{4}} (dx_0 + dx_1 + dx_2 - dx_3)^{\frac{1-r_1-r_2+r_3}{4}} \end{aligned}$$

Here the non-dimensional parameters r_i characterize the rate of anisotropy. If we take the simplest case $r_i = 0$ and introduce the new coordinates $\xi_i = A_{ij} x_j$, where

$$A_{ij} = \begin{pmatrix} 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix} \tag{17}$$

then, the interval Eq.(16) takes the form

$$ds' = ds_{BM} = (d\xi_1 d\xi_2 d\xi_3 d\xi_4)^{1/4} \tag{18}$$

The difference of this approach from the standard theory is the following: the spontaneous symmetry break is accompanied not by the appearance of the cosmological constant, but by the appearance of the space-time anisotropy.

The similar expression for the metrics which factually uses the notion of a volume was used in [4] to construct the theory of gravitation. In [5] independently of [29] there was obtained the conclusion that it is impossible to observe the effects pointing at the difference in metrical properties between Riemann space-time and Finsler space-time on the Solar system scale.

In [28] and the subsequent series of papers the Berwald-Moor metrics is connected with the fundamental mathematical properties of the little known number-like object hyper complex numbers H4. The use of the H4 or other algebra of the kind might lead to the change in the description of phenomena not only in mega scale but on a micro scale of quantum phenomena, and this has a ring with the ideas [34].

It should be underlined that though there are certain promising perspectives in the theory dealing with Finsler geometry, the connection of this direction with observations is insufficient. Moreover, the experiment that could make it possible to judge upon the geometrical properties of space-time has not been suggested up to now.

4. Optic-metrical parametric resonance

As it was mentioned in the previous Section, the experiments dealing with static case don't suite, thats why the GW were suggested as a proper effect to study the space-time geometrical properties. But all the methods suggested up to now to detect the GW (eighteen in number [16]) deal with the registration of the GW effects as the first order perturbations. For the Solar system it means the accuracy of 10^{-24} which is not yet achieved in spite of long lasting efforts and expensive projects. And even in case of success, the extremely small value of the supposed effect would give a small confidence in the results while the problems of registering and processing would be hard to overcome if one intends to use this effect for further investigations.

Let us take the semi-classical model to describe the interaction between the atom and the electromagnetic field which is well known in theoretical spectroscopy [41]. We are going to apply it to describe the action of the GW on the atom of a space maser.

Let us first regard a two-level atom in the monochromatic quasi-resonant strong field with frequency, Ω, which is close to the atomic frequency ω. 'Strong' field means that the stimulated transitions dominate. This system is described in terms of the density matrix on the one hand, and the field is described classically, on the other hand. As a result we get a system of Bloch equations for the density matrix components

$$\frac{d}{dt}\rho_{22} = -\gamma\rho_{22} + 2i\alpha_1 \cos(\Omega t - k_1 y)(\rho_{21} - \rho_{12}) \tag{19}$$

$$(\frac{\partial}{\partial t} + v\frac{\partial}{dy})\rho_{12} = -(\gamma_{12} + i\omega)\rho_{12} - 2i\alpha_1 \cos(\Omega t - k_1 y)(\rho_{22} - \rho_{11}) \tag{20}$$

$$\rho_{22} + \rho_{11} = 1 \tag{21}$$

Here ρ_{22} and ρ_{11} are the populations of the levels, ρ_{12} and ρ_{21} are the polarization terms, γ and γ_{12} are the longitudinal and transversal decay rates of the atom (since level 1 is the ground level, $\gamma_{12} = \gamma/2$); $\alpha_1 = \frac{\mu E}{\hbar}$ is the Rabi parameter (Rabi frequency) proportional to the intensity of the electromagnetic wave (EMW), μ is the dipole momentum, E is the electric stress; $\hbar = 1.05 \cdot 10^{-27}$ erg·s is Planck's constant; $k_1$is the wave vector of the EMW, v is the atom velocity along the Oy-axis pointing at the detector, $\gamma << \alpha_1$ is the condition of the strong field.

In the series of papers [20], [39], [40] the phenomenon of the optic-mechanical parametric resonance was theoretically investigated. If a component of the velocity of such a two-level atom parallel to the wave-vector of the field varies periodically with time at frequency related to the Rabi frequency, then the scattered radiation obtains the so called non-stationary component at the frequency close to the frequency of the atomic transition. In other words, the signal at this frequency will be periodically amplified and attenuated with the frequency of the mechanical oscillations of the atom. This effect is due to the redistribution of the energy between the frequencies due to the parametric resonance. In regular observations this signal cant be registered because of the time averaging, but if a special device known in spectroscopy as a gate detector is used, or the signal is registered in time domain and then processed in a special way, then the non-stationary component can be detected and measured. It turns out that the amplitude of such signal is comparable to the height of the regular peak characterizing the interaction between the atom and the resonant field. That is this signal is large.

Turning to the investigation of the astrophysical system, we see that the sources of the monochromatic EMW are known in space. These are the space masers whose atoms are in the ground states and the transitions take place from the metastable levels, i.e. in this case they fit the two-level model. The saturated space masers realize the conditions of the strong field. On the other hand, we can suppose that there exists the reason due to which the distance and consequently the atom velocity component in the direction at the detector on the Earth would periodically change. This reason is the action of the periodical GW emitted by a pulsar located as shown on Fig.2.

The GW acts on the atomic levels, on the EMW of the maser and on the atom location. In [36] it was shown that the first effect is negligibly small in comparison with the other two. The action of the GW on the monochromatic

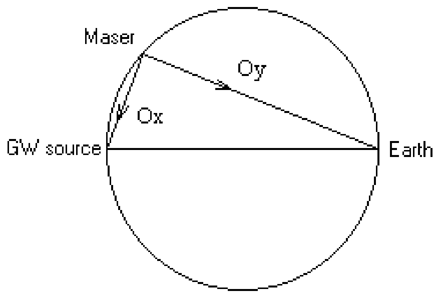

FIGURE 2.

EMW is accounted for when solving the eikonal equation

$$g^{ik}\frac{\partial\psi}{\partial x^i}\frac{\partial\psi}{\partial x^k}=0 \tag{22}$$

The law of the atomic motion must be obtained from the solution of the geodesic equation

$$\frac{d^2x^i}{ds^2}+\Gamma^i_{kl}\frac{dx^k}{ds}\frac{dx^l}{ds}=0 \tag{23}$$

saturated space maser in the field of the GW. Solving them and demanding the conditions of the parametric resonance to be fulfilled, we can calculate the signal. This effect is of the zero order and its detection on the Earth is possible with the help of the already existing radio telescopes that are able to detect the space maser signal.

Such experiment can be used to investigate the space-time geometrical properties in the following way. The theoretical expressions that must be compared to the experiment results should be obtained with the help of the various suggestions about the space-time geometry. The suggestion that gives the best fit with the experimental results will correspond to the geometrical properties of real space-time.

5. Isotropic perturbation of the Minkowski metrics

Let us consider the geometry to be Riemannian and use the regular Einstein equations in the approximation of the weak field far from masses $g^{ik}=g^{(0)ik}+h^{ik}$. The corrections to the metric tensor of the flat space-time suffice the wave equation. In the simplest case for the plane waves it has the form

$$(\frac{\partial^2}{\partial x^2}-\frac{1}{c}\frac{\partial^2}{\partial t^2})h^k{}_i=0 \tag{24}$$

The solution can be naturally taken as [3]

$$h^k{}_i=Re[A^k{}_i\exp(ik_\alpha x^\alpha)] \tag{25}$$

that suffice the equation if $k_\alpha k^\alpha = 0$, i.e. k^α is a light-like vector. That is why the metric tensor can be written as

$$g^{ik} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 + h\cos\frac{D}{c}(x^0 - x^1) & 0 \\ 0 & 0 & 0 & -1 - h\cos\frac{D}{c}(x^0 - x^1) \end{pmatrix} \tag{26}$$

where h is the dimensionless amplitude of the GW ($h << 1$), D is the frequency of the GW.

Solving Eq.(22) with regard to Eq.(26), we see that the GW leads to the phase modulation of the EMW. Since h is small, the latter can be presented [37] as a superposition

$$E(t) = E\cos(\Omega t - ky) + E\frac{\omega}{4D}h[\cos((\Omega - D)t - ky) - \cos((\Omega + D)t - ky)] \tag{27}$$

The solution of Eq.(23) with regard to the Eq.(26) gives [36]

$$y(t) \sim h\frac{c}{D}\sin(Dt + k_g x) \tag{28}$$

where k_g is the wave vector of the GW. The expression Eq.(28) gives the following formula for the component of the atomic velocity in the direction of the Earth

$$v = v_0 + v_1 \cos Dt \tag{29}$$

$$v_1 = hc \tag{30}$$

Substituting Eq.(29) and Eq.(27) into Eq.(19), we get

$$\begin{aligned} \frac{d}{dt}\rho_{22} = -\gamma\rho_{22} + 2i[\alpha_1\cos(\Omega t - ky) + \alpha_2\cos((\Omega - D)t - ky) \\ - \alpha_2\cos((\Omega + D)t - ky)](\rho_{21} - \rho_{12}) \end{aligned} \tag{31}$$

$$\begin{aligned} \frac{d}{dt}\rho_{12} = -(\gamma_{12} + i\omega)\rho_{12} - 2i[\alpha_1\cos(\Omega t - ky) + \alpha_2\cos((\Omega - D)t - ky) \\ - \alpha_2\cos((\Omega + D)t - ky)](\rho_{22} - \rho_{11}) \end{aligned} \tag{32}$$

$$\rho_{22} + \rho_{11} = 1$$

where $\alpha_2 = \frac{\omega h}{4D}\alpha_1$, and where the relation (29) was taken into account in the expression for the full time derivative $\frac{d}{dt} = \frac{\partial}{\partial t} + kv$. The system (31) can be solved by the asymptotic expansion method. If certain conditions on the parameters are fulfilled, we can speak of the optic-metrical parametric resonance (OMPR). These conditions formulated in [36], [37]] have the form:

- The EMW is spectroscopically strong

$$\frac{\gamma}{\alpha_1} = \Gamma\varepsilon; \Gamma = O(1); \varepsilon << 1 \tag{33}$$

- The amplitude condition of the OMPR related to the trichromatic field

(34) $$\frac{\alpha_2}{\alpha_1} = \frac{\omega h}{4D} = a\varepsilon; a = O(1); \varepsilon << 1$$

- The amplitude condition of the OMPR related to the periodic change of the atomic velocity

(35) $$\frac{kv_1}{\alpha_1} = \frac{\omega h}{\alpha_1} = \kappa\varepsilon; \kappa = O(1); \varepsilon << 1$$

- The frequency condition of the OMPR

(36) $$(\omega - \Omega + kv_0)^2 + 4\alpha_1^2 = D^2 + O(\varepsilon) \Rightarrow D \sim 2\alpha_1$$

If the conditions (33-36) are fulfilled, then solving Eqs.(31) by the asymptotic expansion method for the small parameter, ε, we get the principal term of the expansion for $Im(\rho_{21})$ which characterizes the scattered energy flow. At the frequency shifted by D from the central peak, the flow is proportional to ε^0 and has the form

(37) $$Im(\rho_{21}) \sim \frac{\alpha_1}{D} \cos 2Dt + O(\varepsilon)$$

The negative values correspond to the amplification, the positive values correspond to the attenuation of the energy flow at the mentioned frequency due to the redistribution of the energy of the maser radiation in the conditions of the OMPR. Similarly to [20], [39], [40] in the regular observations of the space maser signal it is impossible to observe the non-stationary component because of the time averaging, but the use of the gate detector, or the appropriate processing of the signal in the time domain would provide the observable OMPR signal. This result means that the GW whose existence follows from the GRT and is indirectly supported in [18] could be observed in the direct way with the help of the OMPR based method. More detailed discussion and the analysis of the feasibility of the OMPR conditions for the real astrophysical systems are given in [36], [37]. Here we will only mention that if this type of a signal is detected in the process of purposeful observations, the reason for it can be undoubtedly identified as the GW.

If the GW emitted by the pulsars and the short-period doubles do exist, the OMPR based method can become the foundation of the gravitational astronomy for the inner region of the Milky Way disk. Appendix 1 contains the coordinates of the astrophysical systems suitable for observations both for the galactic vicinity of the Sun and for the periphery of our galaxy (see pulsar 3) which also belongs to the class of spiral galaxies.

But it could happen that the signal in the proposed experiment would be absent or would differ from the predicted one for some of the observation points. This will mean that some essential factors were not taken into account. And the space-time geometrical properties are among these factors.

6. Anisotropic perturbation of the Minkowski metrics

All the calculations leading to Eq.(24) could be repeated, if we change the expression for the metrics to

$$g_{ij}(x) \to g_{ij}(x, \dot{x}) = \eta_{ij}(x) + h_{ij}(x, \dot{x}) \tag{38}$$

where

$$\eta_{ij}(x) = \eta^{(0)}_{ij}(x) \tag{39}$$

is the Minkowski metrics for the flat space, $h_{ij}(x, \dot{x})$ is a small perturbation such that $h^k_i(x, \dot{x}) = \eta^{(0)ki} h_{ij}(x, \dot{x})$. The structure of Einstein equations will remain the same and the perturbation $h^k_i(x, \dot{x})$ will still suffice the wave equation similar to Eq.(24). But the expression Eq.(25) will look like

$$h^k{}_i(x, \dot{x}) = Re[A^k{}_i(\dot{x}) \exp(ik_\alpha x^\alpha)] \tag{40}$$

This means that the amplitude of the GW will vary in various directions of their propagation. From the point of view of observations based on the OMPR method this difference can not be observed directly since the effect is of the zero order. But it will reveal itself in the indirect way, for example, the conditions of the OMPR will be sufficed at different distances in different directions from one and the same GW source. Then the conditions (34) and (35) will transform to the following

- The amplitude condition of the OMPR related to the trichromatic field

$$\frac{\alpha_2 \varsigma_1(\dot{x})}{\alpha_1} = \varsigma_1(\dot{x}) \frac{\omega h}{4Dz'} = a\varsigma_1(\dot{x})\varepsilon; a = O(1); \varepsilon << 1 \tag{41}$$

- The amplitude condition of the OMPR related to the periodic change of the atomic velocity

$$\frac{kv_1 \varsigma_2(\dot{x})}{\alpha_1} = \varsigma_2(\dot{x}) \frac{\omega h}{\alpha_1} = \kappa\varsigma_2(\dot{x})\varepsilon; \kappa = O(1); \varepsilon << 1 \tag{42}$$

Here functions $\varsigma_1(\dot{x}), \varsigma_2(\dot{x})$ are related to the expressions for the amplitudes, $A^k{}_i(\dot{x})$, of the GW.

7. Investigations of the space-time properties with the help of the OMPR effect

In this Section we will analyze the possible results of the OMPR based experiment with regard to the problems mentioned in Sections 1 and 2. It was found [36], [37] that the distances between the GW sources (pulsars or doubles) and space masers are not small but are of interstellar scale. This means that one and the same GW source could affect several masers. Such a source is a kind of a beacon with the frequency now known to the eight decimal digits, while this or that maser is a receiver. The Milky Way scale experiment should be performed in the following way. Let us chose the GW sources in various places of our galaxy and regard several masers together with each of them paying attention to the conditions (33-36). Then we will try to detect the OMPR signal according to the method described in [37] for the GW sources closer to the inner part of the galaxy (IPG) and for the GW sources closer to the periphery part of the galaxy (PPG). One may check that these experiments can give only nine possible outcomes that will have the meanings given below.

1: no OMPR signal for all the masers corresponding to the IPG GW sources, no OMPR signal for all the masers corresponding to the PPG GW sources.

Interpretation: no gravitational waves (and no possibility for the GW astronomy) → Einstein equations for the empty space dont have the structure of the wave equation → no need for dark matter → Riemannian geometry suits.

Problems: choice of the scalar in the variation principle, interpretation of the results in [18].

2: OMPR signal is present for all the masers corresponding to the IPG GW sources, no OMPR signal for all the masers corresponding to the PPG GW sources.

Interpretation: scale dependence (possibly, conformal gravity outside the galaxy), Riemannian geometry suits, GRT in the IPG where GW astronomy is possible.

3: no OMPR signal for all the masers corresponding to the IPG GW sources, OMPR signal is present for all the masers corresponding to the PPG GW sources.

Interpretation: scale dependence (possibly, conformal gravity in the IPG), Riemannian geometry suits, GRT outside the galaxy where GW astronomy is possible.

4: OMPR signal is present for all the masers corresponding to the IPG GW sources, OMPR signal is present for all the masers corresponding to the PPG GW sources.

Interpretation: Riemannian geometry suits, GRT works and GW astronomy is possible.

Problems: dark matter problem.

The rest corresponds to the situation when we have to use Eqs.(41-42) instead of Eqs.(34-35), that is only some of the selected masers behave as they should when the OMPR conditions are fulfilled. This will point at the anisotropy effects mentioned in Section 5.

5: OMPR signal is present for some of the masers corresponding to the IPG GW sources, OMPR signal is present for some of the masers corresponding to the PPG GW sources.

Interpretation: Finslerian geometry suits, GW astronomy is possible.

Problems: dark matter problem.

6: OMPR signal is present for some of the masers corresponding to the IPG GW sources, no OMPR signal for all the masers corresponding to the PPG GW sources.

Interpretation: scale dependence (possibly, conformal gravity outside the galaxy), Finslerian geometry suits in the IPG where GW astronomy is possible.

7: no OMPR signal for all the masers corresponding to the IPG GW sources, OMPR signal is present for some of the masers corresponding to the PPG GW sources.

Interpretation: scale dependence (possibly, conformal gravity in the IPG), Finslerian geometry suits in the PPG where GW astronomy is possible.

8: OMPR signal is present for all the masers corresponding to the IPG GW sources, OMPR signal is present for some of the masers corresponding to the PPG GW sources.

Interpretation: Riemannian geometry suits in the IPG, Finslerian geometry suits in the PPG, GW astronomy is possible.

Problems: dark matter problem.

9: OMPR signal is present for some of the masers corresponding to the IPG GW sources, OMPR signal is present for all the masers corresponding to the PPG GW sources.

Interpretation: Finslerian geometry suits in the IPG, Riemannian geometry suits in the PPG, GW astronomy is possible.

Problems: dark matter problem.

The coordinates of the pairs of masers corresponding to the GW sources located both in the IPG (pulsar 6) and in the PPG (pulsar 7) are given in Appendix 2.

If in the observations we find that the situations 5-9 are realized, then the systematic observations interpreted with the help of expression (40) could give function in the expression for the metrics corresponding to Eq.(14)

(43)

$$g_{ij}(x,\dot{x}) = \eta_{ij}^{(0)}(x) + h_{ij}(x,\dot{x}) = \eta_{ij}^{(0)}(x) + \frac{1}{2}\frac{\partial^2 F^2(x,\dot{x})}{\partial \dot{x}^k \partial \dot{x}^i} = \eta_{ij}^{(0)}(x) + \eta_{kj}^{(0)}(x) h_i^k(x,\dot{x})$$

Thus, if the space-time anisotropy takes place on the galactic scale, then its quantitative characteristic could be obtained in OMPR based experiment.

8. Berwald-Moor metrics

The natural continuation of this approach is the consideration of the situation when the space-time anisotropy is not a small correction as in the previous Section but is described by Finsler geometry. In accord with the experimental approach dealing with the GW described above, one should again use the small linear correction for the empty space, but the unperturbed metrics now is not the Minkowski one, but some Finsler space metrics

$$g_{ij}(x,\dot{x}) = h_{ij}(x,\dot{x}) + \chi_{ij}(x,\dot{x}) \quad (44)$$

It seems appealing to choose the Berwald-Moor metrics for the unperturbed metrics. To speak about the OMPR effect, one should find out explicitly if the GW are possible in such a space-time and write down the corresponding correction to the metrics; then also find out how the description of the electromagnetic processes Eq.(27) change and write down the geodesics equation.

One could expect that the structure of Einstein equations remains that of the wave equation and, thus, the GW would be possible though maybe become more complicated. The geodesics equation seems also to become more complicated, but its solution will still present a technical problem. But the description of the electromagnetic processes and the description of the GW-EMW interaction will present a different kind of a problem.

An essential feature discovered and underlined in [29] is the following: the notion of simultaneity which is the base of any relativistic theory might belong not to the causal structure but to the structure of Lagrangean. This remark causes a profound methodological problem. The choice of Riemann geometry for the description of space-time is closely connected with the invariance of Maxwell equations – the foundation of the majority of experiments. It was this fact that Einstein considered while formulating the relativity principle and while constructing the SRT. Rejecting Riemann geometry, we reject the Maxwell equations invariance, and this means the appearance of the terms that have the metric origin. These we will have to interpret in frames of the known phenomenology. The analogous problem was posed in the end of [38]. The situation becomes even more complicated, if we consider the relation between the gravitation and electromagnetism both for classical GRT effects such as light bending and gravitational red shift and for the direct transformation of gravity and electromagnetism into each other [17]. Finally, we see that the transfer to Finsler geometry demands a detailed physical consideration.

9. Conclusion

The goal of this paper was to suggest an experiment suitable for the investigation of the space-time geometrical properties and to give the corresponding theory. The physical effect underlying such experiment is the optic-metrical parametric resonance described in Sections 3-5 and in papers [36], [37]. The possible results of the OMPR based observations analyzed in Section 6 could give an answer to the question which geometry suits best for the description of the physical space-time. Moreover, these results could also be used to choose the direction of the further fundamental research. If it turns out that Riemann geometry is suitable in the galaxy scale, then astrophysics will confront either the problem of the choice of the variation principle scalar lying in the base of the axiomatic theory, or the problem of the dark matter which has to be solved in frames of the elementary particles theory (and corresponding experiment). In the last case the GW astronomy can appear and be developed. If it turns out that the geometry must be modified and, for example, must become Finsler one, then instead of the mentioned problems the foundations of the electrodynamics must be carefully examined, and this might have far going consequences on all the levels from quantum mechanics to cosmology.

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11. Appendix 1

Coordinates and parameters of the astrophysical systems suitable for the OMPR based detection of the GW [2]-[1]

	Name	RaJ	DecJ	d(pc)	D(Hz)
1.Pulsar	J1022+1001	10:22:58.006	$+10^0 01'52.8''$	300	60.7794489280
Maser	AF Leo	11:25:16.4	$+15^0 25'22''$	270	
2.Pulsar	B0656+14	06:59:48.134	$+14^0 14'21.5''$	290	2.59813685751
Maser	U ORI	05:52:51.0	$+20^0 10'06.0''$	280	
3.Pulsar	J0538+2817	05:38:25.0632	$+28^0 17'9.07''$	1770	6.9852763480
Maser	HH 4	05:37:21.8	$+23^0 49'24.0''$	1700	
4.Pulsar	B0031-07	00:34:08.86	$-07^0 21'53.4''$	720	1.0605004987
Maser	U CET	02:31:19.6	$-13^0 22'02.0''$	660	
5.Double	RXJ0806.3+1527	08:06.3	$+15^0 27''$	100	0.00311526
Maser	RT Vir	13:00:06.1	$+05^0 27'14''$	120	

12. Appendix 2

Coordinates and parameters of the astrophysical systems suitable for the OMPR based detection of the GW (space-time anisotropy test) [2]-[1]

	Name	RaJ	DecJ	d(pc)	D(Hz)
6.Pulsar	J1908+0734	19:08:17.01	$+07^0 34'14.36''$	580	4.70914721426
Maser-1	IRC+10365	18:34:59.0	$+10^0 23'00.0''$	500	
Maser-2	RT AQL	19:35:36.0	$+11^0 36'18.0''$	530	

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