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A SUSTAINABLE JOINT ECONOMIC LOT SIZE MODEL WITH CARBON FOOTPRINT

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Abstract. The paper focus on how the carbon costs modify the classical joint economic lot sizing model of Bannerjee (1986). A number of papers investigate carbon emission of firms in the Economic Order Quantity (EOQ) context. We attempt to classify the most important types of activities leading to carbon emissions, and build them into the model as endogenous factors. We analyze the effects of introducing carbon emission in the model as endogenous variable by employing comparative static analysis. Our results suggest that carbon costs may significantly modify the EOQ ordering policy. In literature there are a limited number of contributions to combine carbon costs vulnerability with logistics and Supply Chain Management (SCM) costs.

1. Introduction

Undoubtedly, sustainability has become one of the key factors of competitiveness in recent years. Issues such as mitigating carbon and GHG emissions, reducing wastes and implementing other reverse logistics related measurements have been more and more in focus not only in theory but also in practice of supply chain management.

Rosen and Kishawy in [19] defined sustainability as the interaction of three main areas; environmental, social and economic factors can be considered related to this topic. However, these sets are not disjunctive; a social-environmental issue is e.g. protection of environment and natural resources; a social-economic can be the corporate responsibility (business ethics, etc.); and economic-environmental tools are: subsidies/incentives and taxes/penalties to promote efficiency and environmental stewardship.

In order to incorporate all of these different factors and impacts, the term: green supply chain management has been created, which reflects all the various aspects of sustainability in supply chains [27].

A key approach in green supply chain management is to elaborate the possibilities of reducing carbon emissions that has been implemented by many leading companies (e.g. Wal-Mart) as Hoffman stated in [11]. Al-Aomar and Weriakat [1] highlighted that a

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very important objective of lean and green supply chain management is to cut CO2 emissions in transport and distribution. Except for practical considerations, many – practice related - examples can be found in the scientific literature that aim to integrate the environmental impact in determining the optimal production-shipment policy of a company by considering the fixed and variable carbon emission costs [24]. Ji [14] analyzed strategies for production and distribution to mitigate carbon footprint. Plambeck [17] proved that operations management and supply chain management can be effective to profitably reduce CO2 emissions.

The paper focus on how the carbon costs modify the classical joint economic lot sizing model of Bannerjee [4]. This model investigates supplier-buyer relationship in a supply chain context.

A number of papers investigate carbon emission of firms in the EOQ context. These papers introduce transportation costs (Bonney, Jaber [7], Wahab et al. [24]), restrictions (Hua et al. [12]), or price discount-type limitations (Bouchery et al. [8]) in the model in order to identify carbon sensitive costs. Our paper contributes to the research line followed by the above mentioned previous works. We attempt to classify the most important types of activities leading to carbon emissions, and build them into the model as endogenous factors. We analyze the effects of introducing carbon emission in the model as endogenous variable by employing comparative static analysis.

The paper will present an EOQ type optimization model involving carbon costs. Our results suggest that carbon costs may significantly modify the EOQ ordering policy. We provide estimations on the difference in EOQ with and without built in carbon costs. Introduction of carbon costs into the EOQ type optimization model will result in lower emission, but higher supply chain costs. The paper provides an abstract picture of real business practices. As mathematical modeling generally, it allows the investigation of limited number of factors. In literature there are a limited number of contributions to combine carbon costs vulnerability with logistics and SCM costs.

In the literature some authors apply a modified EOQ (economic order of quantity) model to determine proper options of mitigating the carbon emission of a firm or a whole supply chain. The basic of these models is generally the joint total relevant cost (JTRC) and joint economic lot size (JELS) approach published by Banerjee (1986). Banerjee created a solution for joint economic lot size problem in a single purchaser-single vendor case, the demand rates and lead times were deterministic as a precondition. Since its publication, the basic model has been extended and amended by sustainability related constraints by many authors. Chen et al [10] used a carbon-constrained EOQ to examine the trade-off between relative reduction in emissions and relative increase in costs. Various regulations types appeared in their paper, such as strict carbon caps, carbon tax, cap-and-offset, cap-and-price.

Battini et al [5] set up a sustainable EOQ-model with the consideration of incorporating carbon footprint management into worldwide firms' business decisions. They applied environmental issues as objectives and not as constraints in the problem. Deterministic product demand, exogenous product price were the preconditions amended by the assumption that the buyer only decides the order size in the business process. The selected methodology was the direct accounting approach. The carbon footprint approach is popular among many other authors e.g. Kannan et al [15], Hua et al [12], Benjafaar et al [6].

Bouchery et al [8] reformulated the original EOQ as a multi-objective problem, and proposed an interactive procedure to select the best regulatory policy to control carbon emissions. The model integrated carbon emission of delivery and also inventory (e.g. refrigerating of products at stock). Three criteria were taken into account for SOQ (sustainable order of quantity) model: carbon footprint, fixed amount of carbon emissions per order, injury rate at ordering and warehousing operations.

Hua et al [12] employed the possibility of carbon emission trade in inventory management. They assumed that product demand is deterministic, the retail price is exogenous and the retailer only decides the order size. They shed light on the dependency of logistic carbon emission; three drivers were identified: mode of transport, fuel used and distance traveled. Zhang and Xu [26] also considered carbon emission trading but with carbon cap. Stochastic demand was assumed and the right of the participants to buy or sell to emit carbon. A profit-maximization model was created for multi-item production planning problem - for single period – with carbon cap and trade mechanism.

Depending on the linearity of objective functions, linear and non-linear inventory EOQ models can be distinguished in the literature. Benjafaar et al [6], Chen et al [10], Bouchery et al [8], Tsai and Yeh [21] examined linear objective functions. There are existing models that applied non-linear objective functions for holding cost, holding emission, transport cost, transport emission or for all of these. Rezaei and Davoodi [18] used non-linear objective function for transport cost, while Pan et al [16] applied non-linear objective function both for transport cost and transport emission. Borogi et al [9] considered non-linear objective functions not only for holding cost and emission; but also for transport cost and emission.

In many supply chain operations in practice, the demand and lead time are uncertain. Thus the EOQ or SOQ models have to be modified because of the stochastic characteristics of these issues. Arikan et al considered transport lead time variability for inventory on carbon emissions. In their model, the retailer determined also the time and the size of the orders. A simulation model was constructed for the problem with the preconditions of stationary demand with non-significant trend and seasonality.

Another interesting point is to integrate the possibility of manufacturing rework to the created models of sustainable production, inventory and distribution. Anutarya et al [2] elaborated a model for manufacturing with rework applying carbon emission constraints aiming profit maximization.

Wang and Choi [25] suggested a different approach: in opposition with the many previous models, the objective in their point of view was not minimizing costs or maximizing

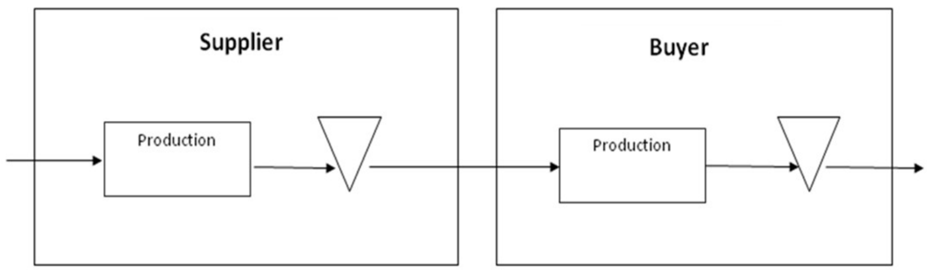

Figure 1. Material flow of the model

profit, but to maximize shareholders' wealth under the conditions of stochastic demand for multi-item manufacturing.

Our paper is organized as follows. The basic model is presented in section 2. In this section we summarize the basic notation of the model with the relevant decision variables and parameters. In section 3 we show the optimal decision of the buyer and supplier. These are not new results, but they present the structure of the optimal behavior of the parties. In the next section we introduce the carbon constraints in this joint economic lot size. In section 5 we construct the optimal joint economic lot size under carbon constraints. We have shown that the EOQ-type carbon constraints can be reduced to two-sided inequalities. The last part of the paper summarizes the results and draws attention to some generalizations of the basic model.

2. The Basic Model

Parameters of the model:

- D: demand of the buyer per time unit,
- P: production rate of the supplier,
- s_b: setup cost of an order for the buyer,
- h_b: holding costs for the buyer,
- s_v: setup cost of an order for the supplier,
- h_v: holding costs for the supplier,
- t^c: length of a cycle.

Decision variables of the model:

- q_b: lot size (order level) for the buyer,
- q_v: lot size (production level) for the supplier,
- q: joint lot size for the system.

The material flow of the model is shown in Figure 1.

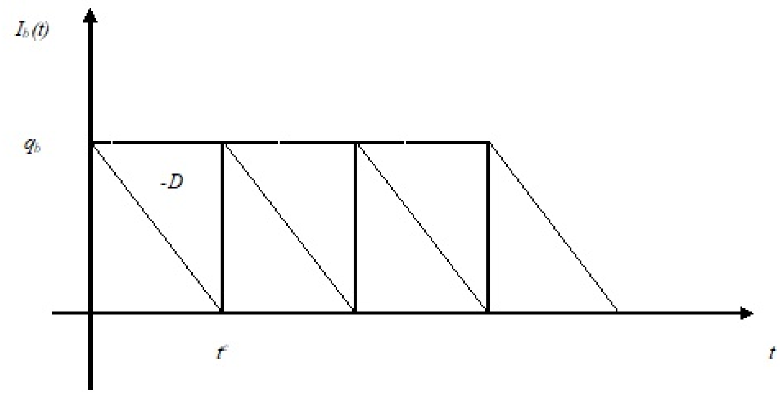

FIGURE 2. Inventory levels of the buyer

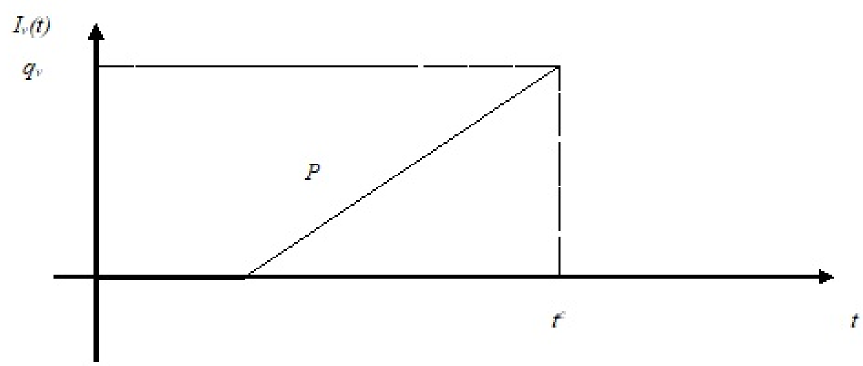

FIGURE 3. Inventory levels of the supplier

In the model we use the traditional assumption of the economic order quantity model (Banerjee [4]). The costs of the supplier and buyer consist of setup cost and holding cost of used and new products. The inventory levels of the buyer and the supplier of the supply chain are shown in Figures 2 and 3.

The cost function of the buyer is

$$TC_b(q_b) = s_b \cdot \frac{D}{q_b} + \frac{q_b}{2} \cdot h_b.$$

The cost function of the supplier is the following in dependence of cost parameters

$$TC_v(q_v) = s_v \cdot \frac{D}{q_v} + \frac{q_v}{2} \cdot h_v.$$

3. The optimal lot sizes and collection rates of the partial models

Banerjee [4] has investigated the development of the costs for both participants of the supply chain. His conclusion is that the total system costs can be reduced with cooperation. However, he did not discuss how to divide the cost savings. Sucky [20] has offered a bargaining model which gives a solution to this problem. Our model is an extension of model of Banerjee [4] with carbon constraints. We summarize the partial and system wide solution of the extended model.

3.1. **Optimal decision of the buyer.** The buyer's problem is a simple EOQ model. The optimal solution for the lot size is

$$q_b^o = \sqrt{\frac{2 \cdot s_b \cdot D}{h_b}},$$

and the optimal costs

$$TC_b(q_b^o) = \sqrt{2 \cdot D \cdot s_b \cdot h_b}.$$

In the next subsection we solve the problem of the supplier without coordinated lot size.

3.2. **Optimal lot size and collection rate of the supplier.** As shown before the solution of the supplier's problem depends on the sequence of the manufacturing and remanufacturing activities.

$$q_v^o = \sqrt{\frac{2 \cdot D \cdot s_v}{h_v}},$$

and the cost function

$$TC_v(q_v^o) = \sqrt{2 \cdot D \cdot s_v \cdot h_v}.$$

In the next section we introduce the carbon emission limit.

4. Carbon constraints of the participants

Let us assume that the carbon emissions of ordering and inventory holding are known. The parameters are the following (Hua et al [12], Chen et al [10]):

- e_b the amount of carbon emission in executing an ordering of the buyer,
- g_b the amount of carbon emission in holding of the products of the buyer,
- C_b carbon cap of the buyer,
- e_v the amount of carbon emission in executing an ordering of the supplier,
- g_v the amount of carbon emission in holding of the products of the supplier,
- C_v carbon cap of the supplier.

The carbon constraints for the buyer are

$$e_b \cdot \frac{D}{q_b} + \frac{q_b}{2} \cdot g_b \leq C_b,$$

and for the supplier

$$e_v \cdot \frac{D}{q_v} + \frac{q_v}{2} \cdot g_v \leq C_v.$$

These two last constraints can be reformulated in the next way

$$\underline{q}_b \leq q_b \leq \overline{q}_b,$$

for the buyer and

$$\underline{q}_v \leq q_v \leq \overline{q}_v,$$

for the supplier, if the inequalities can be solved for some positive lot sizes. The last two inequalities define the set of the possible joint economic lot sizes.

5. The joint economic lot size models with carbon constraints

The joint economic lot size with carbon constraints is with cost function of the buyer

$$TC_b(q) = s_b \cdot \frac{D}{q} + \frac{q}{2} \cdot h_b$$

and the cost function of the supplier is the following in dependence of cost parameters

$$TC_v(q) = s_v \cdot \frac{D}{q} + \frac{q}{2} \cdot h_v$$

such that

$$\max\left\{\underline{q}_b, \underline{q}_v\right\} \leq q \leq \min\left\{\overline{q}_b, \overline{q}_v\right\}.$$

The last inequalities are the carbon constraints for a possible joint economic lot size. If the set of possible joint lot sizes is empty, i.e.

$$\min\left\{\overline{q}_b, \overline{q}_v\right\} < \max\left\{\underline{q}_b, \underline{q}_v\right\}$$

then there exists no solution for the carbon constrained joint economic lot size model.

Let us now analyze the joint economic lot size model with joint cost function

$$TC_b(q) = (s_b + s_v) \cdot \frac{D}{q} + \frac{q}{2} \cdot (h_b + h_v)$$

The joint economic lot size under carbon constraints is

$$q^\circ = \begin{cases} \max\left\{\underline{q}_b, \underline{q}_v\right\}, & \text{if } \sqrt{\frac{2\cdot D\cdot(s_b+s_v)}{(h_b+h_v)}} \leq \max\left\{\underline{q}_b, \underline{q}_v\right\} \\ \sqrt{\dfrac{2\cdot D\cdot(s_b+s_v)}{(h_b+h_v)}}, & \text{if } \max\left\{\underline{q}_b, \underline{q}_v\right\} < \sqrt{\frac{2\cdot D\cdot(s_b+s_v)}{(h_b+h_v)}} < \min\left\{\overline{q}_b, \overline{q}_v\right\} \\ \min\left\{\overline{q}_b, \overline{q}_v\right\}, & \text{if } \min\left\{\overline{q}_b, \overline{q}_v\right\} \leq \sqrt{\frac{2\cdot D\cdot(s_b+s_v)}{(h_b+h_v)}} \end{cases}.$$

The optimal costs are

$$TC_b(q^\circ) = (s_b + s_v) \cdot \frac{D}{q^\circ} + \frac{q^\circ}{2} \cdot (h_b + h_v).$$

The optimal joint economic lot size under carbon constraints differs only in lower and upper restrictions from the classical lot size.

6. Conclusions and further research

In this paper we have investigated an Economic Order Quantity model with carbon constraints. We have given a necessary condition for existence of an optimal solution of the model. We have shown that the system-wide optimal joint economic lot size is influenced by the carbon caps of the participants of the supply chain.

In a next analysis we can investigate an emission trading between the buyer and supplier. If it is allowed to purchase pollution rights, then the supply chain is an open system with market connection. These cases can be modeled with game theoretical models and methods.

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