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WAVELET TRANSFORM AS POWERFUL TOOL TO ANALYSES OF ACOUSTIC EMISSION SIGNALS *

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Abstract. The discrete wavelet transformation is a relative new tool to analyse discrete time series. It is similar to the fast Fourier transformation, however, the basically function is not sine and cosine functions. It decomposes an input vector into approximation (low frequency) and detail (high-frequency) levels. Wavelet analysis is a promising set of tools and techniques for signal analysing of seismic shaking, human speech, financial data, music and many others types of especially non-stationary signals.

Key words. wavelet transform, impact-echo method, acoustic emission, de-noising, time-frequency domain

AMS subject classifications. 15A15, 15A09, 15A23

1. Introduction. Like the fast Fourier transform (FFT), the discrete wavelet transformation (DWT) is a fast, linear operation that operates on a data vector whose length is an integer power of two, transforming it into a numerically different vector of the same length. Also like the FFT, the wavelet transformation is invertible and in fact orthogonal - the inverse transformation, when viewed as a big matrix, is simply the transpose of the transformation. Both FFT and DWT, therefore, can be viewed as a rotation within function space, from the input space (or time) domain, where the basis functions are the unit vectors e_i , or Dirac delta functions in the continuum limit, to a different domain. For the Fourier transform, this new domain has basis functions that are the familiar sinus and cosines. In the wavelet domain, the basis functions are somewhat more complicated and have the fanciful names "mother functions" and "wavelets."

2. Theory. Making-up and definition mother wavelet begins by calculation of dilation equation from reason of obtaining scaling equation. Scaling equation is used for definition mother wavelet. Other wavelet functions are defined by shifting and stretching of mother wavelet. Definition of scaling function and mother wavelet is given by equations (2.1) and (2.2)

$$\phi(x) = \sum_{k=0}^{M-1} c_k \phi(2 \cdot x - k) \tag{2.1}$$

$$\Psi(x) = \sum_{k=0}^{M-1} (-1)^k \cdot c_k \cdot \phi(2 \cdot x + k - M + 1) \tag{2.2}$$

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where $\phi(x)$ is scaling function, $\Psi(x)$ is mother wavelet, k is order of wavelet, c_k are nonzero coefficients defined mother wavelet, M is number of nonzero coefficients c_k. Corresponding mother wavelet must satisfy the following conditions (2.3) till (2.6):

$$\sum_{k=0}^{M-1} c_k = 2 \tag{2.3}$$

$$\sum_{k=0}^{M-1} (-1)^k \cdot k^m \cdot c_k = 0 \qquad m = 0, 1, 2, \cdots, \frac{M}{2} - 1 \tag{2.4}$$

$$\sum_{k=0}^{M-1} c_k \cdot c_{k+2m} = 0 \qquad m = 1, 2, \cdots, \frac{M}{2} - 1 \tag{2.5}$$

$$\sum_{k=0}^{M-1} c_k^2 = 2 \tag{2.6}$$

For example, for coefficient of frequently used Daubeshies wavelet db4, we can solve fours coefficient

$$c_0 = \frac{1+\sqrt{3}}{4} \qquad c_1 = \frac{3+\sqrt{3}}{4} \tag{2.7}$$

$$c_2 = \frac{3-\sqrt{3}}{4} \qquad c_3 = \frac{1-\sqrt{3}}{4} \tag{2.8}$$

The equations (2.9) and (2.11) illustrate the wavelet transformation process by using mother wavelet db4. The discrete wavelet transformation is defined by a square matrix of filter coefficients, transforming an array into a new array of the same length. It was discovered that the wavelet transformation could be implemented with a specifically designed pair of finite impulse response (FIR) filters called quadrature mirror filter pair (QMF). Transform matrix is applied to input (f_1 till f_{16}) yielding smooth data ($A_1 \ldots A_8$) interleaved with detail data ($D_1 \ldots D_8$). The results are permuted to separate the smoothed or decimated data, which must be separated (procedure (2.11)). The detail data is simply stored while the transform matrix is applied to the smoothed data. Each repetition of the process divides the smooth data in halves. The process can be terminated at any point, but usually proceeds until there are only two data points left. This process is often called Malat's pyramidal algorithm.

$$
(2.9)\, C = \left(\begin{array}{cccccccccccccc}
c_0 & c_1 & c_2 & c_3 & & & . & & & & & & \\
c_3 & -c_2 & c_1 & -c_0 & & & . & & & & & & \\
 & & c_0 & c_1 & c_2 & c_3 & . & & & & & & \\
 & & c_3 & -c_2 & c_1 & -c_0 & . & & & & & & \\
 & & & & c_0 & c_1 & . & & & & & & \\
 & & & & c_3 & -c_2 & . & & & & & & \\
 & & & & & & . & & & & & & \\
 & & & & & & . & & & & & & \\
 & & & & & & . & c_2 & c_3 & & & & \\
 & & & & & & . & c_1 & -c_0 & & & & \\
 & & & & & & . & c_0 & c_1 & c_2 & c_3 & & \\
 & & & & & & . & c_3 & -c_2 & c_1 & -c_0 & & \\
 & & & & & & . & & & c_0 & c_1 & c_2 & c_3 \\
 & & & & & & . & & & c_3 & -c_2 & c_1 & -c_0 \\
c_2 & c_3 & & & & & . & & & & & c_0 & c_1 \\
c_1 & -c_0 & & & & & . & & & & & c_3 & -c_2
\end{array}\right)
$$

$$
(2.10) \qquad C \cdot \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \\ f_9 \\ f_{10} \\ f_{11} \\ f_{12} \\ f_{13} \\ f_{14} \\ f_{15} \\ f_{16} \end{pmatrix} = \begin{pmatrix} A_1 \\ D_1 \\ A_2 \\ D_2 \\ A_3 \\ D_3 \\ A_4 \\ D_4 \\ A_5 \\ D_5 \\ A_6 \\ D_6 \\ A_7 \\ D_7 \\ A_8 \\ D_8 \end{pmatrix}
$$

$$
\begin{pmatrix} A_1 \\ D_1 \\ A_2 \\ D_2 \\ A_3 \\ D_3 \\ A_4 \\ D_4 \\ A_5 \\ D_5 \\ A_6 \\ D_6 \\ A_7 \\ D_7 \\ A_8 \\ D_8 \end{pmatrix}
\overset{Permutate}{\Longrightarrow}
\begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \end{pmatrix}
\overset{Transform}{\Longrightarrow}
\begin{pmatrix} \dot{A}_1 \\ \dot{D}_1 \\ \dot{A}_2 \\ \dot{D}_2 \\ \dot{A}_3 \\ \dot{D}_3 \\ \dot{A}_4 \\ \dot{D}_4 \\ \dot{A}_5 \\ \dot{D}_5 \\ \dot{A}_6 \\ \dot{D}_6 \\ \dot{A}_7 \\ \dot{D}_7 \\ \dot{A}_8 \\ \dot{D}_8 \end{pmatrix}
\Longrightarrow \cdots \tag{2.11}
$$

3. Experimental Application. This time-frequency transformation was used for analysis of response at testing structure. The "Impact-Echo" method is known as non-direct acoustic emission method changed up the quality of specimen. Some scientists of the world in this time calls this method as the Resonant Inspection. The specimen in the method is loaded by impulse shock and then the response of testing structure was collected. In this example there were two piezoelectric sensors each on one end of a rod. The rod was made from Wolfram Carbide Cobalt and it has diameter, 5 mm, and length, 60 mm. One sensor transformed electrical impulse width, 5 ms, with period, 8 ms, to mechanical shocks. The other sensor picked up responses. Because tested samples, as it is known, have their own frequencies. The changes of these frequencies mark some differences of tested samples. It is necessary to know spectral characteristic of samples without defects to diagnostic of defects. When it is an interest of ultrasonic area then it is possible to find small discontinuities into samples.

A part of time history of amplitude of the measured original signal with two generated impulses contains mechanical and electrical noise at test (Fig. 3.1). Note that scientists assume this noise as white. Its frequency spectrum (Fig. 3.2) computed by Fourier transformation has main frequency on 80 Hz. This signal was de-noised (Fig. 3.3) by using discrete wavelet transformation. The db 4 wavelet base was chosen from a grouf of tested ones as usable. The soft minimaxi thresholding and re-scaling done using level-dependent estimation of level noise mode was set-up for de-noising the signal. The de-noised signal (Fig. 3.3) with comparing to the original signal (Fig. 3.1) is put near to the theoretical signal. The frequency spectrum of de-noised signal (Fig. 3.3) contents the same frequency components as the original signal spectrum (Fig. 3.4). That means that this method cut off only spurious signal.

In Figs. 3.5 and 3.6 there are shown the response signals from above described experiment. Each of figures contains signal and its decomposition into five approximated (to the left) and five detailed (to the right) levels. The wavelet decomposition by Daubeschies 4 was applied. The original recorded signal is shown in Fig. 3.5 and de-noising of this signal is in Fig. 3.6. The advantages of this transformation are evident already on the first decomposition level (as approximation so details).

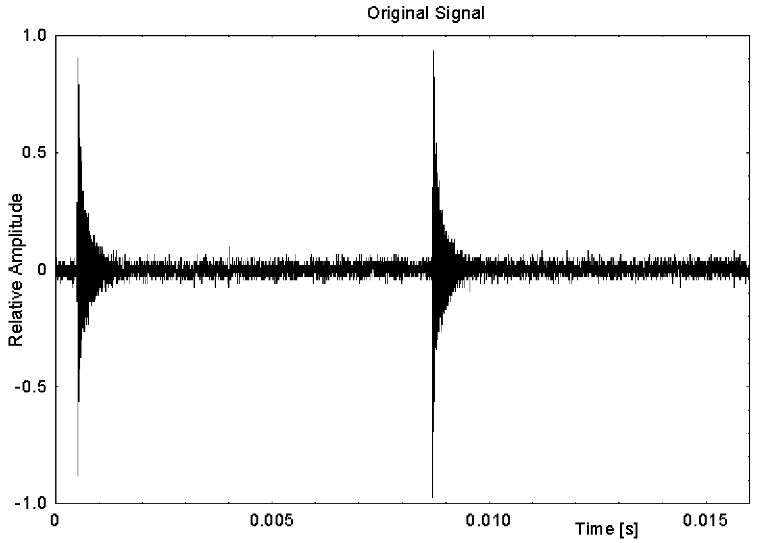

FIG. 3.1. *The recorded signal at testing.*

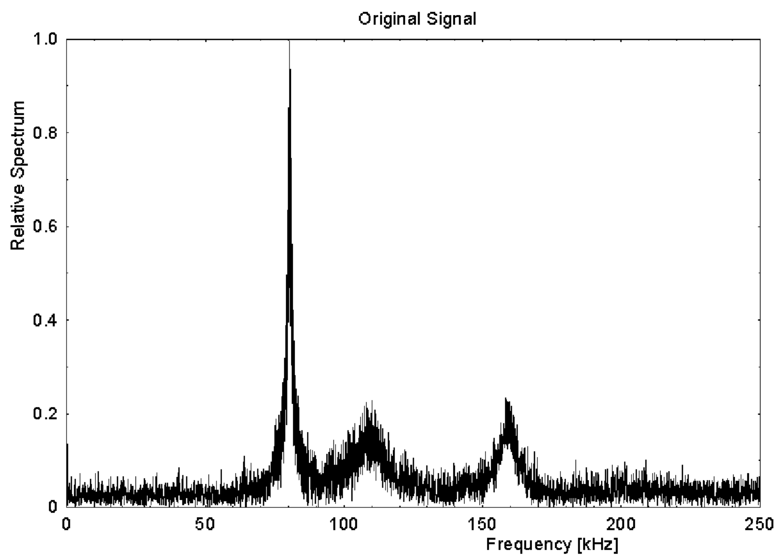

FIG. 3.2. *The frequency spectrum of original signal.*

It is useful to note that selected response is noised as electrical as mechanical noise. The wavelet transformation is one of good tool to de-noise these signals. Its advantage in this example is that based function is not sinusoidal. This transformation eliminates only noise amplitudes.

As mother wavelet it was chosen Doubechies 4 because its parameters are better than other tested wavelet base (see Tab. 3.1). We note as a noise signal in this table there was cut a part of signal from 2 ms to 8 ms. To obtain the root-mean-square (RMS) value, use norm(A)/sqrt(n). Note that norm(A), where A is an n-element vector, is the length of A.

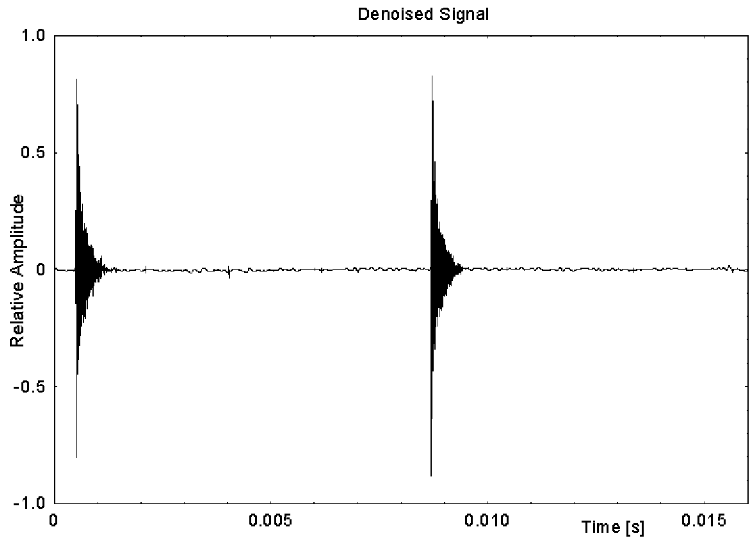

FIG. 3.3. *The de-noised signal by Discrete Wavelet Transformation.*

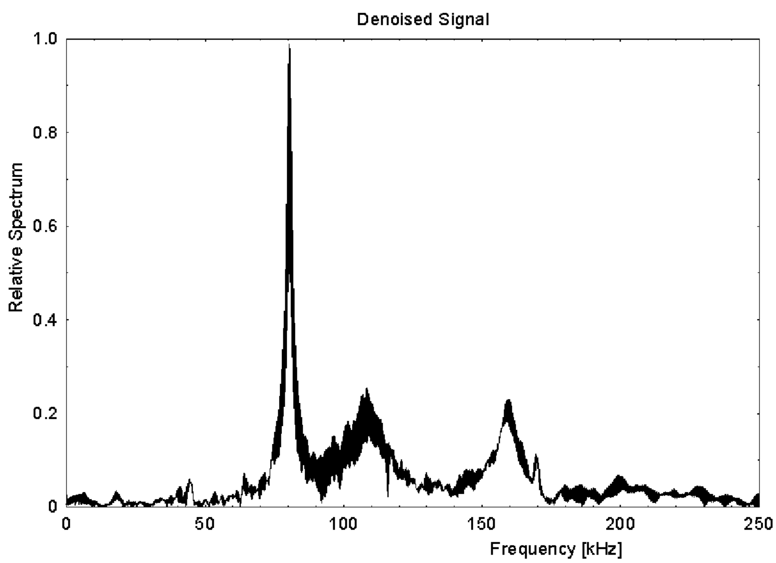

FIG. 3.4. *The frequency spectrum of the de-noised signal.*

4. Conclusion. Discrete wavelet transformation provides a new approach for studying discrete time series. The following examples describe just a small sample of what researchers can do with wavelets. In substance, the potential applications of discrete wavelet transformation can be divided into six fields:

- detection of long-term evolution
- detection of self-similarity
- identifying pure frequencies
- de-noising signals
- compressing signals
- suppressing signals

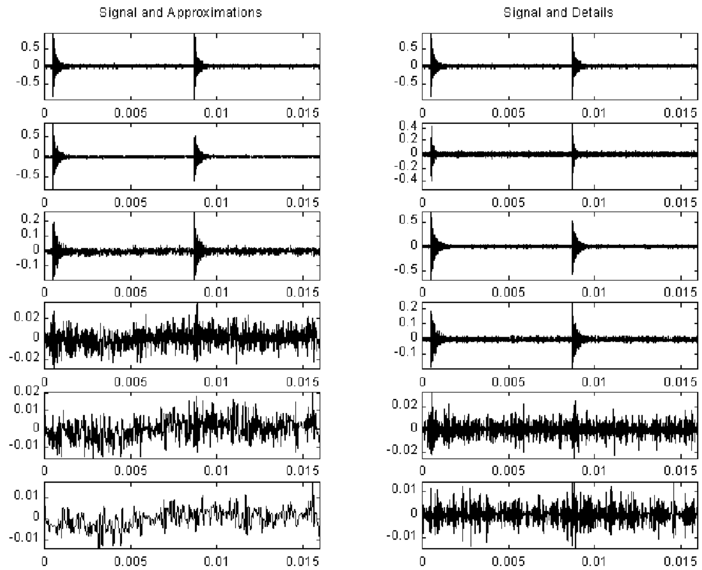

FIG. 3.5. *Decomposition of the original signal to 5th level by db4*

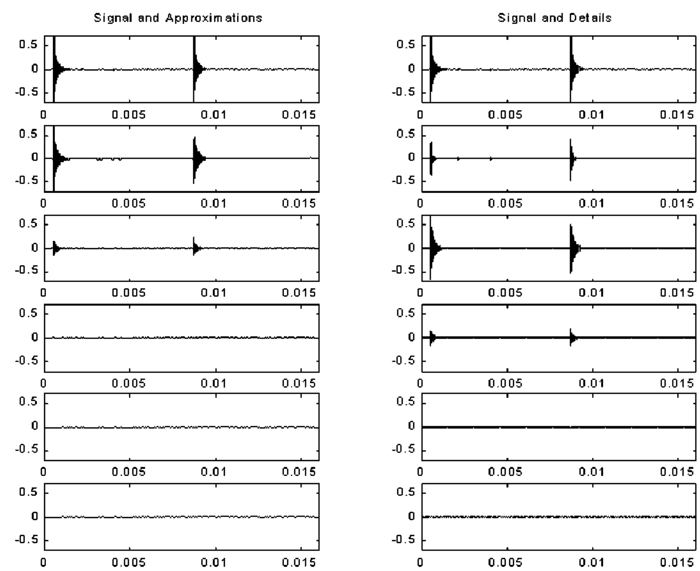

FIG. 3.6. *Decomposition of the de-noised signal to 5th level by db4*

They are some advantages wavelet transformation compared to traditional Fourier methods in analysing physical situations where the signal contains discontinuities and sharp spikes.

In this case the tested measured signal - composed from the periodical ensued responses on shock and the noise signals - was de-noised by application discrete wavelet transformation. Because the basic function of wavelet (mother wavelet) is not harmonic function as Fourier transform has, the frequency spectrum was not too modified.

TABLE 3.1
Test of some mother Wavelet to the signal de-noising.

Mother wavelet	RMS_{noise} to $RMS_{noise_{none}}$	RMS_{noise} $\times 10^{-5}$	RMS_{signal} $\times 10^{-4}$	RMS_{signal} to $RMS_{signal_{none}}$	RMS_{signal} to RMS_{noise}
none	1	39.4531	6.4830	1	1.6432
dB1	0.2137	8.4311	5.1767E	0.7985	6.1400
dB2	0.2132	8.4114	5.0840	0.7842	6.0442
dB3	0.2120	8.3640	5.0477	0.7786	6.0350
dB4	0.2014	7.9458	5.0561	0.7799	6.3632
dB5	0.2118	8.3561	5.0814	0.7838	6.0810
dB6	0.2143	8.4548	5.1054	0.7875	6.0384
dB7	0.2082	8.2141	5.1281	0.7910	6.2430
dB8	0.2106	8.3088	5.1475	0.7940	6.1952
dB9	0.2067	8.1549	5.1605	0.7960	6.3280
sym1	0.2132	8.4114	5.0840	0.7842	6.0442
sym2	0.2120	8.3640	5.0477	0.7786	6.0350
sym3	0.2037	8.0366	5.0561	0.7799	6.2914
sym4	0.2064	8.1431	5.1118	0.7885	6.2775
sym5	0.2086	8.2299	5.1157	0.7891	6.2160
coif1	0.2072	8.1747	5.0470	0.7785	6.1740
coif2	0.2075	8.1865	5.1177	0.7894	6.2514
coif3	0.2105	8.3049	5.1611	0.7961	6.2146
coif4	0.2131	8.4074	5.1870	0.8001	6.1696
coif5	0.2135	8.4232	5.2007	0.8022	6.1742
bior1.1	0.2132	8.4114	5.0840	0.7842	6.0442
bior2.2	0.2265	8.9361	5.0075	0.7724	5.6036
bior3.3	0.2698	10.6442	4.9349	0.7612	4.6361
bior4.4	0.2052	8.0958	5.1086	0.788	6.3102
bior5.5	0.2054	8.1036	5.1559	0.7953	6.3625

Acoustic emission signals are not from group of harmonic signals. Therefore the basis of the wavelet transform is near to presume signal than the basis of Fourier transform.

Note that Fourier transform is not suitable for analysis of non-stationary signal as burst type of acoustic emission ones is.

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