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SOME MODELS FOR THE VOLTAGE

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Abstract

The aim of this paper is to give a model for the voltage generated in the ultrasonic cavitation.

1 Preliminaries

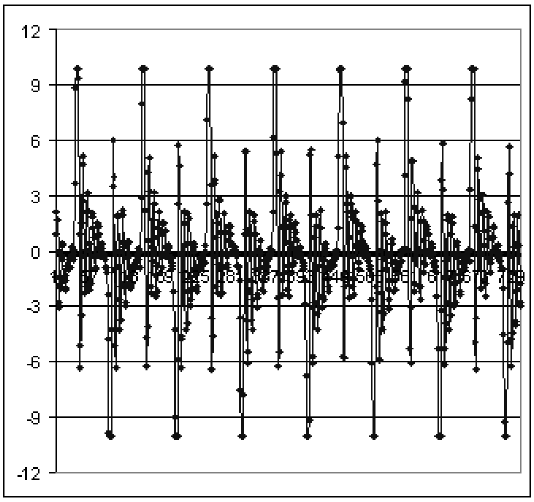

Figure 1: The signal form

Key Words: time series; autocorelation function.

In order to study the voltage appeared in the ultrasonic cavitation, we used an data acquisition card. The data obtained are represented in the figure 1.

We shall use the following:

Definition 1. *A time series is a realization of a stochastic process or a sequence of values that shows the volume variation of a statistic population or of the characteristic's level, related to the time.*

Definition 2. *A discrete time process is a sequence of random variables* $(X_t; t \in \mathbf{Z})$.

Definition 3. *A discrete time process* $(X_t; t \in \mathbf{Z})$ *is stationary if*:

$$(\forall) t \in \mathbf{Z}, M(X_t^2) < \infty, \tag{1}$$

$$(\forall) t \in \mathbf{Z}, M(X_t) = \mu, \tag{2}$$

$$(\forall) t \in \mathbf{Z}, (\forall) h \in \mathbf{Z},\ Cov(X_t,\ X_{t+h}) = \gamma(h). \tag{3}$$

where M(X) is the expectation value of X *and Cov(X, Y) is the correlation of the variables* X *and* Y.

Definition 4. *A stationary process* $(\varepsilon_t; t \in \mathbf{Z})$ *is called a white noise if* $\gamma(h) = 0$, *for* $h \neq 0$, $M(\varepsilon_t) = 0$ *and* $D^2(\varepsilon_t) = \sigma^2 = \gamma(0)$, $(\forall) t \in \mathbf{Z}$.

Definition 5. *The function defined on* $\mathbf{Z}$, *by*:

$$\rho(h) = \frac{Cov(X_t, X_{t+h})}{\sqrt{D^2(X_t) D^2(X_{t+h})}} = \frac{\gamma(h)}{\gamma(0)}$$

is called the autocorrelation function (ACF) of the process $(X_t; t \in \mathbf{Z})$.

Definition 6. *If* $(X_t; t \in \mathbf{Z})$ *is a stationary process, the function defined by*:

$$\tau(h) = \frac{Cov(X_t - X_t^*, X_{t-h} - X_{t-h}^*)}{D^2(X_t - X_t^*)},\ h \in \mathbf{Z}_+$$

is called the partial autocorrelation function (PACF), where X_t^* (X_{t-h}^*) *is the affine regression of* X_t (X_{t-h}) *with respect* $X_{t-1}, ..., X_{t-h+1}$.

Definition 7. *Consider*

$$\begin{aligned} B(X_t) &= X_{t-1} \\ \Phi(B) &= 1 - \varphi_1 B - ... - \varphi_p B^p,\ \varphi_p \neq 0, \\ \Theta(z) &= 1 - \theta_1 B - \theta_2 B^2 - ... - \theta_p B^q,\ \theta_p \neq 0 \\ \Delta^d X_t &= (1 - B)^d X_t \end{aligned}$$

The process $(X_t; t \in \mathbf{Z}_+)$ *is called* ARIMA (p, d, q) *if*:

$$\Phi(B) \Delta^d X_t = \Theta(B) \varepsilon_t,$$

where the absolute values of the roots of Φ *and* Θ *are greater than 1 and* $(\varepsilon_t, t \in \mathbf{Z})$ *is a white noise.*

If $d = 0$, *the* ARIMA(p, d, q) *process is an* ARMA(p, q) *process.*

2 The model

Looking to the figure 1, we can see that there exists a periodicity of the signal and after the study of the given values, we determine the period - 0.000105 s.

The horizontal axis is the time axis (in 0.000001s) and the vertical axis is the voltage axis (in V).

So, we shall study only a period.

```
     Auto- Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25   0   .25  .5   .75   1  Q     Prob.
                   +----+----+----+----+----+----+----+----+
  1   .736   .097                  .   I***.***********      57.440  .000
  2   .304   .097                  .   I***.**               67.358  .000
  3  -.049   .096                  .  *I   .                 67.620  .000
  4  -.184   .096                  ****I   .                 71.307  .000
  5  -.103   .095                  . **I   .                 72.466  .000
  6   .069   .095                  .   I*  .                 73.001  .000
  7   .216   .094                  .   I****                 78.256  .000
  8   .250   .094                  .   I***.*                85.395  .000
  9   .158   .093                  .   I***.                 88.265  .000
 10   .003   .093                  .   *   .                 88.266  .000
 11  -.116   .092                  . **I   .                 89.835  .000
 12  -.132   .092                  .***I   .                 91.910  .000
 13  -.050   .091                  .  *I   .                 92.212  .000
 14   .068   .091                  .   I*  .                 92.777  .000
 15   .145   .090                  .   I***.                 95.348  .000
 16   .135   .090                  .   I***.                 97.619  .000

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .
Total cases:  103     Computable first lags:  102
```

Figure 2: The autocorrelation function of the data

```
    Pr-Aut- Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25   0   .25  .5   .75   1
                   +----+----+----+----+----+----+----+----+
  1   .736   .099                  .   I***.***********
  2  -.518   .099            ******.***I   .
  3  -.024   .099                  .   *   .
  4   .112   .099                  .   I** .
  5   .126   .099                  .   I***.
  6   .047   .099                  .   I*  .
  7   .074   .099                  .   I*  .
  8  -.009   .099                  .   *   .
  9  -.056   .099                  .  *I   .
 10  -.040   .099                  .  *I   .
 11   .004   .099                  .   *   .
 12   .029   .099                  .   I*  .
 13   .034   .099                  .   I*  .
 14   .026   .099                  .   I*  .
 15   .001   .099                  .   *   .
 16  -.016   .099                  .   *   .

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .
Total cases:  103     Computable first lags:  102
```

Figure 3: The partial autocorrelation function of the data

In order to determine the type of the process, for the data on a period, we have to study the autocorrelation and the partial autocorrelation functions (ACF and PACF) for the voltage, V.

First, we consider the hypothesis:

$$\mathrm{H}_0\text{: The autocorrelation function for } V \text{ is zero.}$$

In the figures 2 and 3 are given the values of ACF and PACF at the lag 1, 2, ..., 16. In the 5 - th column appear the corresponding values of Box -

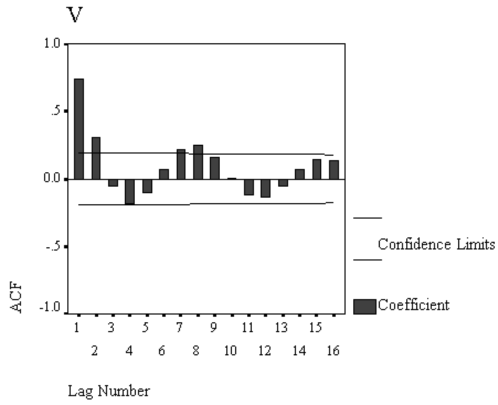

Figure 4: The graph of ACF of the data

Ljung (Q) statistic and in the 6 - th column, the acceptance probabilities for the hypothesis H_0.

Consider the significance level $\alpha = 0.05$.

The values of Box - Ljung statistic must be compared with the values of $\chi^2_{0.05}(k)$, where k is the number of observed data.

In our case, the values of Q - statistic are bigger than $\chi^2_{0.05}(103)$ and the probabilities are closed to zero. So, H_0 is rejected.

In the figures 4 and 5 it can be seen that there are values of ACF and PACF outside the confidence interval, confirming the existence of the autocorrelation for V.

The form of the two graph suggests the existence of an ARMA process.

Here we shall present only the ARIMA(2,0,0) with and without constant.

First, it was considered a model ARIMA(2, 0, 0), including a constant.

After the parameter determination we've obtained:

$\varphi_1 = 1.5615034,\ \varphi_2 = -0.899325,\ const = -0.4272949$.

To test the significance of the parameter's values, the t - test were used. The values obtained for the t -ratio are:

$31.362927,\ -20.095106,\ -1.192045$.

These have to be compared with the value $t_{k-1;0.05} = t_{102;0.05} = 1.96$, from the table of the Student quantiles.

Since $|-1.192045| < 1.96$,the hypothesis that the constant is zero can be accepted. Thus, we make the study for model ARIMA(2, 0, 0)$\equiv$AR(2), without constant.

The new parameters are: $\varphi_1 = 1.5636298,\ \varphi_2 = -0.89193194$and the cor-

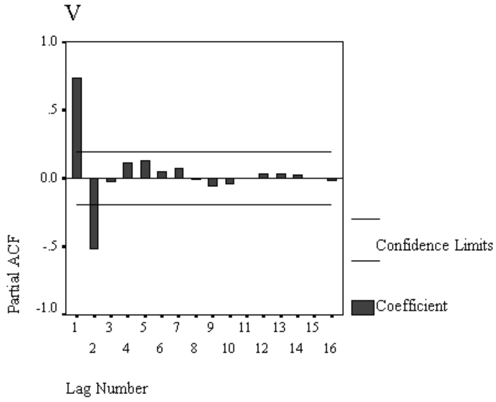

Figure 5: The graph of PACF of the data

responding t - ratios: 31.032973 and -19.394363, with the modulus greater than 1.96. The calculated probabilities to reject the hypotheses that the coefficient are 0 are closed to 1.

Thus, the model is:

$$V_t = 1.5636298 V_{t-1} - 0.89193194 V_{t-2} + \varepsilon_t,\ t \geq 3,$$

where ε_t is the residual.

It remains to prove that the residuals for an white noise.

```
      Auto-  Stand.
Lag   Corr.    Err. -1   -.75   -.5 -.25    0    .25  .5    .75   1    Q    Prob.
                     +----+----+----+----+----+----+----+----+
  1  -.009   .097                     .    *    .                   .008  .927
  2   .021   .097                     .    *    .                   .056  .972
  3   .075   .096                     .    I** .                    .670  .880
  4   .058   .096                     .    I*   .                   1.043 .903
  5   .124   .095                     .    I** .                    2.747 .739
  6   .052   .095                     .    I*   .                   3.053 .802
  7   .110   .094                     .    I** .                    4.422 .730
  8   .082   .094                     .    I** .                    5.182 .738
  9   .063   .093                     .    I*   .                   5.642 .775
 10   .006   .093                     .    *    .                   5.646 .844
 11  -.019   .092                     .    *    .                   5.689 .893
 12  -.005   .092                     .    *    .                   5.692 .931
 13   .010   .091                     .    *    .                   5.705 .956
 14   .040   .091                     .    I*   .                   5.897 .969
 15   .036   .090                     .    I*   .                   6.055 .979
 16   .038   .090                     .    I*   .                   6.234 .985

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .
Total cases:   103     Computable first lags:  102
```

Figure 6: The ACF for the residue

In the figures 6 and 8 it can be seen that the values of the ACF for the residuals are inside the confidence interval and the probabilities of acceptance of the white noise hypothesis are big.

```
   Pr-Aut- Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25   0   .25  .5   .75   1
                   +----+----+----+----+----+----+----+----+
  1 -.009   .099                  .   *   .
  2  .021   .099                  .   *   .
  3  .076   .099                  .   I** .
  4  .060   .099                  .   I*  .
  5  .124   .099                  .   I** .
  6  .050   .099                  .   I*  .
  7  .103   .099                  .   I** .
  8  .068   .099                  .   I*  .
  9  .048   .099                  .   I*  .
 10 -.026   .099                  .  *I   .
 11 -.056   .099                  .  *I   .
 12 -.054   .099                  .  *I   .
 13 -.028   .099                  .  *I   .
 14  .012   .099                  .   *   .
 15  .027   .099                  .   I*  .
 16  .040   .099                  .   I*  .

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .
Total cases:  103     Computable first lags:  102
```

Figure 7: The PACF for the residue

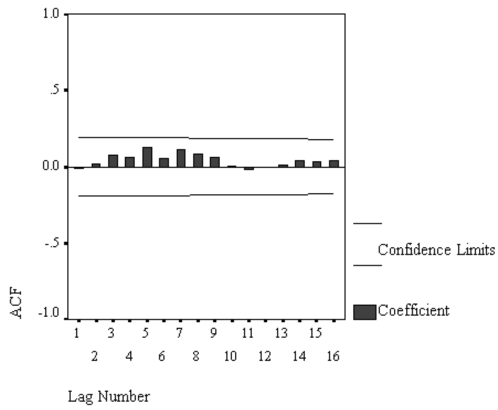

Figure 8: The graph of ACF for the residue

The values of the Q -statistic are less than $\chi^2_{0.05}(100)$.The PACF values are also inside the confidence interval and they are small (see figure 7). Thus, the residuals form a white noise and the model is well chosen.

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