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COMPLETE HYPERGROUPS, 1-HYPERGROUPS AND FUZZY SETS

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Abstract

In this paper we determine the fuzzy grades of the complete hypergroups of order less or equal to 6 and some properties of them.

1 Introduction

The correspondence between fuzzy sets and hyperstructures is known since 90's. In 2002, Corsini [2] has proved that it is possible to consider a new connection between fuzzy sets and hypergroupoids. In the following, we shall use notations, definitions and terminology in Corsini [1].

Given a hypergroupoid $< H, \circ >$, we can associate a membership function $\widetilde{\mu} : H \to [0,1]$ as follows:

$$\begin{aligned} &\forall (x,y) \in H^2,\ \forall u \in H,\ \text{we set} \\ &\quad \mu_{xy}(u) = 0 \text{ if and only if } u \notin x \circ y \\ &\quad \mu_{xy}(u) = \frac{1}{|x \circ y|} \text{ if and only if } u \in x \circ y \\ &\quad A(u) = \sum_{(x,y)\in H^2} \mu_{xy}(u) \\ &\quad Q(u) = \{(a,b) \in H^2 \mid u \in a \circ b\},\ q(u) = |Q(u)| \\ &\quad \widetilde{\mu}(u) = A(u)/q(u). \end{aligned} \tag{ω}$$

From $\widetilde{\mu}$ one obtains a join space ${}^1H$ as follows:

$$\forall (x,y) \in H^2,\ x \circ_1 y = \{z \mid \widetilde{\mu}(x) \wedge \widetilde{\mu}(y) \leq \widetilde{\mu}(z) \leq \widetilde{\mu}(x) \vee \widetilde{\mu}(y)\} \tag{1}$$

Key Words: hypergroup;complete hypergroups; fuzzy set.

Then, from $< {}^1H, \circ_1 >$ we obtain, in the same way, a membership function $\widetilde{\mu}_1$ and we associate the join space ${}^2H$ and so on. A sequence of fuzzy sets and join spaces $< {}^rH, \widetilde{\mu}_r >$ is determined.

Theorem 1.1. *A hypergroup H is a complete hypergroup if and only if has the form $H = \bigcup\limits_{g \in G} A_g$, where:*

(i) *G is a group;*
(ii) *$\forall g_1, g_2 \in G,\ g_1 \neq g_2$, we have $A_{g_1} \cap A_{g_2} = \emptyset$;*
(iii) *if $(a, b) \in A_{g_1} \times A_{g_2}$, then $a \circ b = A_{g_1 g_2}$ (see [1]).*

Definition 1.2. *A hypergroup H is called 1-hypergroup if the cardinality of its core ω_H is 1* (see [1]).

Definition 1.3. *A hypergroupoid H has fuzzy grade $fg(H) = r$ if $\forall i < r$, ${}^iH$ and ${}^{i+1}H$ are not isomorphic and $\forall j > r$, ${}^jH$ is isomorphic to ${}^rH$. We say that H has strong fuzzy grade $sfg(H) = r$ if $fg(H) = r$ and $\forall i > r$, ${}^iH = {}^rH$* (see [3]).

2 Some properties of the complete hypergroup

Definition 2.1. *Let n be a positive integer, $n \geq 3$. A k-decomposition of n, $2 \leq k \leq n-1$, is an ordered system of natural numbers $(m_1, m_2, ..., m_k)$ such that $m_i \geq 1$, $\forall 1 \leq i \leq k$, $m_1 + m_2 + ... + m_k = n$.*

We denote $d_k(n)$ the number of k-decompositions of n.

Theorem 2.2. *There are $\sum\limits_{k=2}^{n-1} s_k d_k(n)$ nonisomorphic complete hypergroups of order n, where s_k denotes the number of the nonisomorphic groups of order k.*

Proof. We fix an arbitrary group G of order k and let H be a complete hypergroup of order n. Since $H = A_{g_1} \cup A_{g_2} \cup ... \cup A_{g_k}$, we obtain $|H| = \sum\limits_{i=1}^{k} |A_{g_i}|$. Hence there are $d_k(n)$ possibilities of making a partition of H as in Theorem 1.1.

It follows that the class of complete hypergroups of order n has the cardinality $\sum\limits_{k=2}^{n-1} s_k d_k(n)$. ∎

Theorem 2.3. *Let H be a complete hypergroup. Then, for $\forall u \in H$, $\widetilde{\mu}(u) = \dfrac{1}{|A_{g_u}|}$, where $u \in A_{g_u}$.*

Proof. By hypothesis, for$\forall u \in H$, there is a unique $g_u \in G$, such that $u \in A_{g_u}$. From (ω) we obtain

$$\forall u \in H,\ A(u) = \sum_{\substack{(x,y)\in H^2\\ u\in x\circ y}} \mu_{xy}(u) = \sum_{\substack{(x,y)\in H^2\\ u\in x\circ y}} \frac{1}{|x\circ y|} = \sum_{\substack{(x,y)\in H^2\\ u\in x\circ y}} \frac{1}{|A_{g_x g_y}|} =$$

$$= \sum_{\substack{(x,y)\in H^2\\ u\in x\circ y}} \frac{1}{|A_{g_u}|} = \frac{q(u)}{|A_{g_u}|}.$$

So, $\widetilde{\mu}(u) = \dfrac{1}{|A_{g_u}|}$.

We used that $u \in x \circ y = A_{g_x g_y}$ and g_u is unique in G with the property $u \in A_{g_u}$ imply that $g_x g_y = g_u$. ∎

Theorem 2.4. *Let H be an arbitrary hypergroup of order n and R the equivalence defined on H: $xRy \Longleftrightarrow \widetilde{\mu}(x) = \widetilde{\mu}(y)$. If $|H/R| = 2$ and each equivalence class has the same cardinality, then ${}^2H$ is a total hypergroup and $sfg(H) = 2$. In this case n must be an even number.*

Proof. Let H be the set $H = \{e, a_1, ..., a_{n-1}\}$, $n = 2k$, $k \in \mathbb{N} \setminus \{0,1\}$. We denote $e = a_0$. Since $|H/R| = 2$, we can write $H/R = C_1 \cup C_2$, $|C_1| = |C_2| = k$. After suitable relabelling of the elements of H if necessary, $C_1 = \{a_0, a_1, ..., a_{k-1}\}$ and $C_2 = \{a_k, a_{k+1}, ..., a_{n-1}\}$, so that $\forall 0 \leq i \leq k-1$, $\hat{a}_i = C_1$, and $\forall k \leq i \leq n-1$, $\hat{a}_i = C_2$.

For any $a_i, a_j \in H$ there are two possibilities:
- $\hat{a}_i = \hat{a}_j$ then $a_i \circ_1 a_j = \hat{a}_i$
- $\hat{a}_o \neq \hat{a}_j$ then $a_i \circ_1 a_j = H$ (by (1)).

The join space ${}^1H$ associated is

${}^1H$	e	a_1	$\cdots$	a_{k-1}	a_k	a_{k+1}	$\cdots$	a_{n-1}
e	C_1	C_1	$\cdots$	C_1	H	H	$\cdots$	H
a_1		C_1	$\cdots$	C_1	H	H	$\cdots$	H
$\vdots$			$\ddots$	$\vdots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
a_{k-1}				C_1	H	H	H	H
a_k					C_2	C_2	$\cdots$	C_2
a_{k+1}						C_2	$\cdots$	C_2
$\vdots$							$\ddots$	$\ldots$
a_{n-1}								C_2

hence $\widetilde{\mu}_1(e) = \widetilde{\mu}_1(a_1) = \cdots = \widetilde{\mu}_1(a_{n-1})$. We conclude that ${}^2H$ is a total hypergroup and ${}^rH = {}^2H$, $\forall r \geq 3$. ■

3 The sequences of fuzzy sets and of join spaces determined by all the complete hypergroups of order less or equal to 6

Theorem 3.1. *Let H be a complete hypergroup of order $n \leq 6$.*

(i) *For $n = 3$, $sfg(H) = 1$.*

(ii) *For $n = 4$, there are 3 hypergroups with $sfg(H) = 1$ and two hypergroups with $sfg(H) = 2$.*

(iii) *For $n = 5$, $sfg(H) = 1$.*

(iv) *For $n = 6$, there are 17 hypergroups of $sfg(H) = 1$ and 4 hypergroups of $sfg(H) = 2$.*

Proof. (i) Let consider $H = \{e, a_1, a_2\}$ a complete hypergroup. In this case $G = (\mathbb{Z}_2, +)$, so we have two structures:

a)

H	e	a_1	a_2
e	e	A_1	A_1
a_1		e	e
a_2			e

where $A_0 = \{e\}$, $A_1 = \{a_1, a_2\}$. We note that H is an 1-hypergroup. By Theorem 2.3, $\widetilde{\mu}(e) = 1$, $\widetilde{\mu}(a_1) = \widetilde{\mu}(a_2) = 0,5$.

The join space ${}^1H$ associated is

${}^1H$	e	a_1	a_2
e	e	H	H
a_1		A_1	A_1
a_2			A_1

hence $\widetilde{\mu}_1(e) = 0,47$, $\widetilde{\mu}_1(a_1) = \widetilde{\mu}_1(a_2) = 0,42$. Then ${}^rH = {}^1H$, $\forall r \geq 2$.

b)

H	e	a_1	a_2
e	A_0	A_0	A_1
a_1		A_0	A_1
a_2			A_0

where $A_0 = \{e, a_1\}$, $A_1 = \{a_2\}$. We have $\widetilde{\mu}(e) = \widetilde{\mu}(a_1) = 0,5$ and $\widetilde{\mu}(a_2) = 1$.

It follows that the join space ${}^1H$ associated is

H	e	a_1	a_2
e	A_0	A_0	H
a_1		A_0	H
a_2			a_2

with $\widetilde{\mu}_1(e) = \widetilde{\mu}_1(a_1) = 0,417$, $\widetilde{\mu}_1(a_2) = 0,467$, so ${}^rH = {}^1H$, $\forall r \geq 2$.

ii) Let $H = \{e, a_1, a_2, a_3\}$ be a complete hypergroup.

a) Setting $G = (\mathbb{Z}_2, +)$ we obtain:

a$_1$)

H	e	a_1	a_2	a_3
e	e	A_1	A_1	A_1
a_1		e	e	e
a_2			e	e
a_3				e

where $A_0 = \{e\},\ A_1 = \{a_1, a_2, a_3\}$. H is an 1-hypergroup. The membership function is $\widetilde{\mu}(e) = 1,\ \widetilde{\mu}(a_i) = 0,(3),\ 1 \leq i \leq 3$.

The join space associated is:

^1H	e	a_1	a_2	a_3
e	e	H	H	H
a_1		A_1	A_1	A_1
a_2			A_1	A_1
a_3				A_1

for which $\widetilde{\mu}_1(e) = 0,36,\ \widetilde{\mu}_1(a_i) = 0,3,\ 1 \leq i \leq 3$, hence $^rH = {}^1H$, $\forall r \geq 2$.

a$_2$)

H	e	a_1	a_2	a_3
e	A_0	A_0	A_1	A_1
a_1		A_0	A_1	A_1
a_2			A_0	A_0
a_3				A_0

where $A_0 = \{e, a_1\},\ A_1 = \{a_2, a_3\}$. We obtain $\widetilde{\mu}(e) = \widetilde{\mu}(a_i),\ 1 \leq i \leq 3$ and it follows that ^1H is a total hypergroup and $sfg(H) = 1$.

a$_3$)

H	e	a_1	a_2	a_3
e	A_0	A_0	A_0	A_1
a_1		A_0	A_0	A_1
a_2			A_0	A_1
a_3				A_0

with $A_0 = \{e, a_1, a_2\},\ A_1 = \{a_3\}$. Then $\widetilde{\mu}(e) = \widetilde{\mu}(a_1) = \widetilde{\mu}(a_2) = 0,(3)$ and $\widetilde{\mu}(a_3) = 1$. The join space associated is isomorphic with the one of a$_1$), hence $^rH = {}^1H$, $\forall r \geq 2$.

b) For $G = (\mathbb{Z}_3, +)$ we distinguish two hypergroups :

b$_1$)

H	e	a_1	a_2	a_3
e	e	a_1	A_2	A_2
a_1		A_2	e	e
a_2			a_1	a_1
a_3				a_1

where $A_0 = \{e\},\ A_1 = \{a_1\},\ A_2 = \{a_2, a_3\}$. H is an 1-hypergroup.

b$_2$)

H	e	a_1	a_2	a_3
e	A_0	A_0	a_2	a_3
a_1		A_0	a_2	a_3
a_2			a_3	A_0
a_3				a_2

where $A_0 = \{e, a_1\},\ A_1 = \{a_2\},\ A_2 = \{a_3\}$.

In the last two cases, using Theorem 2.4, the $sfg(H) = 2$ and ^2H is a total hypergroup.

iii) Set $H = \{e, a_1, a_2, a_3, a_4\}$ a complete hypergroup.

a) There are five complete 1-hypergroups of order 5.

a$_1$)

H	e	a_1	a_2	a_3	a_4
e	e	A_1	A_1	A_1	A_1
a_1		e	e	e	e
a_2			e	e	e
a_3				e	e
a_4					e

where $A_0 = \{e\}$, $A_1 = \{a_i\}$, $1 \le i \le 4$. One obtains $\widetilde{\mu}(e) = 1$, $\widetilde{\mu}(a_i) = 0,25$, $1 \le i \le 4$.

The join space ${}^1H$ associated is

H	e	a_1	a_2	a_3	a_4
e	e	H	H	H	H
a_1		A_1	A_1	A_1	A_1
a_2			A_1	A_1	A_1
a_3				A_1	A_1
a_4					A_1

hence $\widetilde{\mu}_1(e) = 0,29$ and $\widetilde{\mu}_1(a_i) = 0,23$, $1 \le i \le 4$. It follows ${}^rH = {}^1H$, $\forall r \ge 2$.

a$_2$)

H	e	a_1	a_2	a_3	a_4
e	e	a_1	A_2	A_2	A_2
a_1		A_2	e	e	e
a_2			a_1	a_1	a_1
a_3				a_1	a_1
a_4					a_1

with $A_0 = \{e\}$, $A_1 = \{a_1\}$, $A_2 = \{a_2, a_3, a_4\}$. We have $\widetilde{\mu}(e) = \widetilde{\mu}(a_1) = 1$ and $\widetilde{\mu}(a_2) = \widetilde{\mu}(a_3) = \widetilde{\mu}(a_4) = 0,(3)$. Then

${}^1H$	e	a_1	a_2	a_3	a_4
e	e, a_1	e, a_1	H	H	H
a_1		e, a_1	H	H	H
a_2			A_2	A_2	A_2
a_3				A_2	A_2
a_4					A_2

is the join space associated and the membership function is: $\widetilde{\mu}_1(e) = \widetilde{\mu}_1(a_1) = 0,28$, $\widetilde{\mu}_1(a_i) = 0,26$, $2 \le i \le 4$. So ${}^rH = {}^1H$, $\forall r \ge 2$.

a$_3$)

H	e	a_1	a_2	a_3	a_4
e	e	A_1	A_1	A_2	A_2
a_1		A_2	A_2	e	e
a_2			A_2	e	e
a_3				A_1	A_1
a_4					A_1

where $A_0 = \{e\}$, $A_1 = \{a_1, a_2\}$, $A_3 = \{a_3, a_4\}$. We calculate $\widetilde{\mu}(e) = 1$, $\widetilde{\mu}(a_i) = 0,5$, $1 \le i \le 4$. The join space associated is the same as in a$_1$), so ${}^rH = {}^1H$, $\forall r \ge 1$.

a$_4$)

H	e	a_1	a_2	a_3	a_4
e	e	a_1	a_2	A_3	A_3
a_1		a_2	A_3	e	e
a_2			e	a_1	a_1
a_3				a_2	a_2
a_4					a_2

where $A_0 = \{e\}$, $A_1 = \{a_1\}$, $A_2 = \{a_2\}$, $A_3 = \{a_3, a_4\}$. So, $\widetilde{\mu}(e) = \widetilde{\mu}(a_1) = \widetilde{\mu}(a_2) = 1$, $\widetilde{\mu}(a_3) = \widetilde{\mu}(a_4) = 0,5$; the membership function $\widetilde{\mu}_1$ is the same as in a$_2$) and we have again $sfg(H) = 1$.

a$_5$) For $G = (K, \cdot)$ the Klein's group and $A_0 = \{e\}$, $A_1 = \{a_1\}$, $A_2 = \{a_2\}$, $A_3 = \{a_3, a_4\}$, we obtain

H	e	a_1	a_2	a_3	a_4
e	e	a_1	a_2	A_3	A_3
a_1		e	A_3	a_2	a_2
a_2			e	a_1	a_1
a_3				e	e
a_4					e

with the join space ${}^1H$ equal to the one of a$_4$), hence ${}^rH = {}^1H$, $\forall r \geq 2$.

b) The following complete hypergroups are not 1-hypergroups :

b$_1$)

H	e	a_1	a_2	a_3	a_4
e	A_0	A_0	A_1	A_1	A_1
a_1		A_0	A_1	A_1	A_1
a_2			A_0	A_0	A_0
a_3				A_0	A_0
a_4					A_0

where $A_0 = \{e, a_1\}$, $A_1 = \{a_2, a_3, a_4\}$ and $\widetilde{\mu}(e) = \widetilde{\mu}(a_1) = 0,5$, $\widetilde{\mu}(a_2) = \widetilde{\mu}(a_3) = \widetilde{\mu}(a_4) = 0,(3)$. The join space ${}^1H$ is the following:

${}^1H$	e	a_1	a_2	a_3	a_4
e	A_0	A_0	H	H	H
a_1		A_0	H	H	H
a_2			A_1	A_1	A_1
a_3				A_1	A_1
a_4					A_1

which leads to ${}^rH = {}^1H$, $\forall r \geq 1$.

b$_2$)

H	e	a_1	a_2	a_3	a_4
e	A_0	A_0	A_0	A_1	A_1
a_1		A_0	A_0	A_1	A_1
a_2			A_0	A_1	A_1
a_3				A_0	A_0
a_4					A_0

where $A_0 = \{e, a_1, a_2\}$, $A_1 = \{a_3, a_4\}$. $\widetilde{\mu}(e) = \widetilde{\mu}(a_1) = \widetilde{\mu}(a_2) = 0,(3)$, $\widetilde{\mu}(a_3) = \widetilde{\mu}(a_4) = 0,5$.

The join space associated is isomorphic with the one of the case b$_1$), so $sfg(H) = 1$.

b$_3$)

H	e	a_1	a_2	a_3	a_4
e	A_0	A_0	A_0	A_0	a_4
a_1		A_0	A_0	A_0	a_4
a_2			A_0	A_0	a_4
a_3				A_0	a_4
a_4					A_0

with $A_0 = \{e, a_1, a_2, a_3\}$, $A_1 = \{a_4\}$. We have $\widetilde{\mu}(e) = \widetilde{\mu}(a_i) = 0,25$, $1 \leq i \leq 3$, $\widetilde{\mu}(a_4) = 1$. The join space associated is

${}^1H$	e	a_1	a_2	a_3	a_4
e	A_0	A_0	A_0	A_0	H
a_1		A_0	A_0	A_0	H
a_2			A_0	A_0	H
a_3				A_0	H
a_4					a_4

for which $\widetilde{\mu}_1(e) = \widetilde{\mu}_1(a_i) = 0,233$, $1 \leq i \leq 3$, $\widetilde{\mu}_1(a_4) = 0,289$. Hence ${}^rH = {}^1H$, $\forall r \geq 2$.

b$_4$)

H	e	a_1	a_2	a_3	a_4
e	A_0	A_0	a_2	A_2	A_2
a_1		A_0	a_2	A_2	A_2
a_2			A_2	A_0	A_0
a_3				a_2	a_2
a_4					a_2

where $A_0 = \{e, a_1\}$, $A_1 = \{a_2\}$, $A_2 = \{a_3, a_4\}$. We have $\widetilde{\mu}(e) = \widetilde{\mu}(a_i) = 0,5$, $i \in \{1, 3, 4\}$, $\widetilde{\mu}(a_2) = 1$. It follows the join space

${}^1H$	e	a_1	a_2	a_3	a_4
e	H_2	H_2	H	H_2	H_2
a_1		H_2	H	H_2	H_2
a_2			a_2	H	H
a_3				H_2	H_2
a_4					H_2

where $H_2 = H \setminus \{2\}$, which is isomorphic with the preceding one. Then ${}^rH = {}^1H$, $\forall r \geq 1$.

b$_5$)

H	e	a_1	a_2	a_3	a_4
e	A_0	A_0	A_0	a_3	a_4
a_1		A_0	A_0	a_3	a_4
a_2			A_0	a_3	a_4
a_3				a_4	A_0
a_4					a_3

where $A_0 = \{e, a_1, a_2\}$, $A_1 = \{a_3\}$, $A_2 = \{a_4\}$. It leads to the join space from the case b$_1$).

b$_6$)

H	e	a_1	a_2	a_3	a_4
e	A_0	A_0	a_2	a_3	a_4
a_1		A_0	a_2	a_3	a_4
a_2			a_3	a_4	A_0
a_3				A_0	a_2
a_4					a_3

with $A_0 = \{e, a_1\}$, $A_1 = \{a_2\}$, $A_2 = \{a_3\}$, $A_3 = \{a_4\}$. The join space is the one of b$_1$).

b$_7$)

H	e	a_1	a_2	a_3	a_4
e	A_0	A_0	a_2	a_3	a_4
a_1		A_0	a_2	a_3	a_4
a_2			A_0	a_4	a_3
a_3				A_0	a_2
a_4					A_0

with $A_0 = \{e, a_1\}$, $A_1 = \{a_2\}$, $A_2 = \{a_3\}$, $A_3 = \{a_4\}$ and $G = (K, \cdot)$ the Klein's group. Again the join space is the one from b$_1$).

iv) a) The following complete hypergroups

a$_1$)

H	e	a_1	a_2	a_3	a_4	a_5
e	e	a_1	a_2	A_3	A_3	A_3
a_1		a_2	A_3	e	e	e
a_2			e	a_1	a_1	a_1
a_3				a_2	a_2	a_2
a_4					a_2	a_2
a_5						a_2

where $A_0 = \{e\}$, $A_1 = \{a_1\}$, $A_2 = \{a_2\}$, $A_3 = \{a_3, a_4, a_5\}$, $G = (\mathbb{Z}_4, +)$.

a$_2$)

H	e	a_1	a_2	a_3	a_4	a_5
e	A_0	A_0	A_0	a_3	a_4	a_5
a_1		A_0	A_0	a_3	a_4	a_5
a_2			A_0	a_3	a_4	a_5
a_3				a_4	a_5	A_0
a_4					A_0	a_3
a_5						a_4

where $A_0 = \{e, a_1, a_2\}$, $A_1 = \{a_3\}$, $A_2 = \{a_4\}$, $A_3 = \{a_5\}$, $G = (\mathbb{Z}_4, +)$.

a$_3$)

H	e	a_1	a_2	a_3	a_4	a_5
e	e	a_1	a_2	A_3	A_3	A_3
a_1		e	A_3	a_2	a_2	a_2
a_2			e	a_1	a_1	a_1
a_3				e	e	e
a_4					e	e
a_5						e

with $G = (K, \cdot)$ the Klein's group, $A_0 = \{e\}$, $A_1 = \{a_1\}$, $A_2 = \{a_2\}$, $A_3 = \{a_3, a_4, a_5\}$.

a$_4$)

H	e	a_1	a_2	a_3	a_4	a_5
e	A_0	A_0	A_0	a_3	a_4	a_5
a_1		a_0	A_0	a_3	a_4	a_5
a_2			A_0	a_3	a_4	a_5
a_3				A_0	a_5	a_4
a_4					A_0	a_3
a_5						A_0

with $G = (K, \cdot)$ the Klein's group, $A_0 = \{e, a_1, a_2\}$, $A_1 = \{a_3\}$, $A_2 = \{a_4\}$, $A_3 = \{a_5\}$.

have the property $|H/R| = 2$ and all the equivalence classes have the same cardinality. By Theorem 2.4, $sfg(H) = 2$.

b) The other complete hypergroups of order 6 have $sfg(H) = 1$. We list them, without writing the join spaces and the membership functions associated.

b$_1$)

H	e	a_1	a_2	a_3	a_4	a_5
e	e	A_1	A_1	A_1	A_1	A_1
a_1		e	e	e	e	e
a_2			e	e	e	e
a_3				e	e	e
a_4					e	e
a_5						e

where
$A_0 = \{e\}$,
$A_1 = \{a_1, a_2, a_3, a_4, a_5\}$.
H is an 1-hypergroup.

b$_2$)

H	e	a_1	a_2	a_3	a_4	a_5
e	A_0	A_0	A_1	A_1	A_1	A_1
a_1		A_0	A_1	A_1	A_1	A_1
a_2			A_0	A_0	A_0	A_0
a_3				A_0	A_0	A_0
a_4					A_0	A_0
a_5						A_0

where
$A_0 = \{e, a_1\}$,
$A_1 = \{a_2, a_3, a_4, a_5\}$.

b$_3$)

H	e	a_1	a_2	a_3	a_4	a_5
e	A_0	A_0	A_0	A_1	A_1	A_1
a_1		A_0	A_0	A_1	A_1	A_1
a_2			A_0	A_1	A_1	A_1
a_3				A_0	A_0	A_0
a_4					A_0	A_0
a_5						A_0

where $A_0 = \{e, a_1, a_2\}$, $A_1 = \{a_3, a_4, a_5\}$.

b$_4$)

H	e	a_1	a_2	a_3	a_4	a_5
e	A_0	A_0	A_0	A_0	A_1	A_1
a_1		A_0	A_0	A_0	A_1	A_1
a_2			A_0	A_0	A_1	A_1
a_3				A_0	A_1	A_1
a_4					A_0	A_0
a_5						A_0

where $A_0 = \{e, a_1, a_2, a_3\}$, $A_1 = \{a_4, a_5\}$.

b$_5$)

H	e	a_1	a_2	a_3	a_4	a_5
e	A_0	A_0	A_0	A_0	A_0	a_5
a_1		A_0	A_0	A_0	A_0	a_5
a_2			A_0	A_0	A_0	a_5
a_3				A_0	A_0	a_5
a_4					A_0	a_5
a_5						A_0

with $A_0 = \{e, a_1, a_2, a_3, a_4\}$, $A_1 = \{a_5\}$.

b$_6$)

H	e	a_1	a_2	a_3	a_4	a_5
e	e	a_1	A_2	A_2	A_2	A_2
a_1		A_2	e	e	e	e
a_2			a_1	a_1	a_1	a_1
a_3				a_1	a_1	a_1
a_4					a_1	a_1
a_5						a_1

where $A_0 = \{e\}$, $A_1 = \{a_1, a_2, a_3, a_4\}$, $A_1 = \{a_5\}$. H is an 1-hypergroup.

b$_7$)

H	e	a_1	a_2	a_3	a_4	a_5
e	e	A_1	A_1	A_2	A_2	A_2
a_1		A_2	A_2	e	e	e
a_2			A_2	e	e	e
a_3				A_1	A_1	A_1
a_4					A_1	A_1
a_5						A_1

where $A_0 = \{e\}$, $A_1 = \{a_1, a_2\}$, $A_2 = \{a_3, a_4, a_5\}$. H is an 1-hypergroup.

b$_8$)

H	e	a_1	a_2	a_3	a_4	a_5
e	A_0	A_0	a_2	A_2	A_2	A_2
a_1		A_0	a_2	A_2	A_2	A_2
a_2			A_2	A_0	A_0	A_0
a_3				a_2	a_2	a_2
a_4					a_2	a_2
a_5						a_2

where $A_0 = \{e, a_1\}$, $A_1 = \{a_2\}$, $A_2 = \{a_3, a_4, a_5\}$.

b$_9$)

H	e	a_1	a_2	a_3	a_4	a_5
e	A_0	A_0	A_1	A_1	A_2	A_2
a_1		A_0	A_1	A_1	A_2	A_2
a_2			A_2	A_2	A_0	A_0
a_3				A_2	A_0	A_0
a_4					A_1	A_1
a_5						A_1

where $A_0 = \{e, a_1\}$, $A_1 = \{a_2, a_3\}$, $A_2 = \{a_4, a_5\}$.

b$_{10}$)

H	e	a_1	a_2	a_3	a_4	a_5
e	A_0	A_0	A_0	a_3	A_2	A_2
a_1		A_0	A_0	a_3	A_2	A_2
a_2			A_0	a_3	A_2	A_2
a_3				A_2	A_0	A_0
a_4					a_3	a_3
a_5						a_3

where $A_0 = \{e, a_1, a_2\}$, $A_1 = \{a_3\}$, $A_2 = \{a_4, a_5\}$.

b$_{11}$)

H	e	a_1	a_2	a_3	a_4	a_5
e	A_0	A_0	A_0	A_0	a_4	a_5
a_1		A_0	A_0	A_0	a_4	a_5
a_2			A_0	A_0	a_4	a_5
a_3				A_0	a_4	a_5
a_4					a_5	A_0
a_5						a_4

where $A_0 = \{e, a_1, a_2, a_3\}$, $A_1 = \{a_4\}$, $A_2 = \{a_5\}$.

b$_{12}$)

H	e	a_1	a_2	a_3	a_4	a_5
e	e	a_1	A_2	A_2	A_3	A_3
a_1		A_2	A_3	A_3	e	e
a_2			e	e	a_1	a_1
a_3				e	a_1	a_1
a_4					A_2	A_2
a_5						A_2

with $A_0 = \{e\}$, $A_1 = \{a_1\}$, $A_2 = \{a_2, a_3\}$, $A_3 = \{a_4, a_5\}$. $G = (\mathbb{Z}_4, +)$. H is an 1-hypergroup.

b$_{13}$)

H	e	a_1	a_2	a_3	a_4	a_5
e	A_0	A_0	a_2	a_3	A_3	A_3
a_1		A_0	a_2	a_3	A_3	A_3
a_2			a_3	A_3	A_0	A_0
a_3				A_0	a_2	a_2
a_4					a_3	a_3
a_5						a_3

where $A_0 = \{e, a_1\}$, $A_1 = \{a_2\}$, $A_2 = \{a_3\}$, $A_3 = \{a_4, a_5\}$. $G = (\mathbb{Z}_4, +)$.

b$_{14}$)

H	e	a_1	a_2	a_3	a_4	a_5
e	e	a_1	A_2	A_2	A_3	A_3
a_1		e	A_3	A_3	A_2	A_2
a_2			e	e	a_1	a_1
a_3				e	a_1	a_1
a_4					e	e
a_5						e

where $A_0 = \{e\}$, $A_1 = \{a_1\}$, $A_2 = \{a_2, a_3\}$, $A_3 = \{a_4, a_5\}$. $G = (K, \cdot)$ the Klein's group. H is an 1-hypergroup.

b$_{15}$)

H	e	a_1	a_2	a_3	a_4	a_5
e	A_0	A_0	a_2	a_3	A_3	A_3
a_1		A_0	a_2	a_3	A_3	A_3
a_2			A_0	A_3	a_3	a_3
a_3				A_0	a_2	a_2
a_4					A_0	A_0
a_5						A_0

where $A_0 = \{e, a_1\}$, $A_1 = \{a_2\}$, $A_2 = \{a_3\}$, $A_3 = \{a_4, a_5\}$. $G = (K, \cdot)$ the Klein's group.

b$_{16}$)

H	e	a_1	a_2	a_3	a_4	a_5
e	e	a_1	a_2	a_3	A_4	A_4
a_1		a_2	a_3	A_4	e	e
a_2			A_4	e	a_1	a_1
a_3				a_1	a_2	a_2
a_4					a_3	a_3
a_5						a_3

where $A_0 = \{e\}$, $A_1 = \{a_1\}$, $A_2 = \{a_2\}$, $A_3 = \{a_3\}$, $A_4 = \{a_4, a_5\}$. H is an 1-hypergroup.

b$_{17}$)

H	e	a_1	a_2	a_3	a_4	a_5
e	A_0	A_0	a_2	a_3	a_4	a_5
a_1		A_0	a_2	a_3	a_4	a_5
a_2			a_3	a_4	a_5	A_0
a_3				a_5	A_0	a_2
a_4					a_2	a_3
a_5						a_4

where $A_0 = \{e, a_1\}$, $A_1 = \{a_2\}$, $A_2 = \{a_3\}$, $A_3 = \{a_4\}$, $A_4 = \{a_5\}$.

References

[1] P. Corsini, *Introducere în teoria hipergrupurilor*, Ed. Univ. "Al.I. Cuza" Iaşi, 1998.

[2] P. Corsini, *A new connection between hypergroups and fuzzy sets*, accepted by Southest Asian Bulletin of Mathematics, 2002.

[3] P. Corsini, I. Cristea, *Fuzzy grade of i.p.s. hypergroups of order* 7, accepted by Iranian Journal of Fuzzy Systems.

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