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MORE ON MORITA CONTEXTS FOR NEAR-RINGS

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Abstract

In [2], the Morita context has been introduced for near-rings as a generalization of a Morita context for rings. In this paper, we are studying ideals in Morita contexts for near-rings, finding new results which continue those in [3]. Firstly, ideals of a Morita context for near-rings are considered in the two ways, that is, one is along to ideals of the near-rings and of the bigroups; another is along to ideals of the Morita context itself. Ideals when M (or Γ) has a strong unity, prime-ideals and equi-prime ideals are studied obtaining their mutual relationship. To an ideal of a Morita context itself, we put into correspondence another ideal of it. We may construct also a quotient Morita context. An important example of a Morita context which includes $M_n(R)$ (the matrix near-ring over a near-ring R) as an operator ring is given and one ideal of it is obtained. Under certain condition, a Morita context (R, Γ, M, L) is represented by $(R, M^*, M, Map_R(M, M))$, where $M^* = Map_R(M, R)$.

Lastly, we can find a categorical equivalence between R-groups and L-groups, where R and L are near-rings in a Morita context (R, Γ, M, L).

1. Preliminaries

Let R be a right near-ring and let Γ be a group (not necessarily abelian). Γ is a *left R-group* if there is a function $R \times \Gamma \to \Gamma$, $(r, \gamma) \to r\gamma$, such that $(r + s)\gamma = r\gamma + s\gamma$ and $(rs)\gamma = r(s\gamma)$, for all $r, s \in R$, $\gamma \in \Gamma$.

We say that Γ is a *right R-group* if there is a function $\Gamma \times R \to \Gamma$, $(\gamma, r) \to \gamma r$, such that $(\gamma + \lambda)r = \gamma r + \lambda r$ and $\gamma(rs) = (\gamma r)s$, for all $\gamma, \lambda \in \Gamma$ and $r, s \in R$.

Key Words: Morita contents; near-rings; R-groups

If R and L are near-rings and Γ is a group, then Γ is called an *R-L-bigroup* if Γ is both a left R-group and a right L-group such that

$$(r\gamma)l = r(\gamma l)\ ,\ \text{ for all } r \in R,\ l \in L,\ \ \gamma \in \Gamma.$$

Definition 1.1. [see [2] 1.1. Definition]. A quadruple (R, Γ, M, L) is a *Morita context for near-rings,* if R and L are near-rings, Γ and M are groups such that

(i) Γ is an R-L-bigroup;

(ii) M is an L-R-bigroup;

(iii) there exists a function: $\Gamma \times M \to R,\ (\gamma, m) \to \gamma m$, such that

$$(\gamma_1 + \gamma_2)m = \gamma_1 m + \gamma_2 m,\ r(\gamma m) = (r\gamma)m,\ (\gamma m)r = \gamma(mr)$$

and $(\gamma l)m = \gamma(lm)$, for all $\gamma_1, \gamma_2, \gamma \in \Gamma,\ m \in M,\ r \in R,\ l \in L$;

(iv) there exists a function: $M \times \Gamma \to L,\ (m, \gamma) \to m\gamma$, such that

$$(m_1 + m_2)\gamma = m_1\gamma + m_2\gamma,\ l(m\gamma) = (lm)\gamma,\ (m\gamma)l = m(\gamma l)$$

and $(mr)\gamma = m(r\gamma)$, for all $m_1, m_2, m \in M, \gamma \in \Gamma,\ r \in R,\ l \in L$;

(v) the two above functions are connected by

$$\gamma_1(m\gamma_2) = (\gamma_1 m)\gamma_2 \ \text{ and } \ (m_1\gamma)m_2 = m_1(\gamma m_2),$$

for all $\gamma_1, \gamma_2, \gamma \in \Gamma,\ m_1, m_2, m \in M$.

Note that:

$$(m_1\gamma_1)(m_2\gamma_2) = ((m_1\gamma_1)m_2)\gamma_2 = (m_1(\gamma_1 m_2))\gamma_2 =$$
$$= m_1((\gamma_1 m_2)\gamma_2) = m_1(\gamma_1(m_2\gamma_2)).$$

Similarly, we have

$$(\gamma_1 m_{12})(\gamma_2 m_2) = ((\gamma_1 m_1)\gamma_2)m_2 = (\gamma_1(m_1\gamma_2))m_2 = \gamma_1((m_1\gamma_2)m_2) =$$
$$= \gamma_1(m_1(\gamma_2 m_2)),\ \text{for all }\ m_1, m_2 \in M,\ \gamma_1, \gamma_2 \in \Gamma.$$

Due to *(v),* we can represent the above products by $\gamma_1 m\gamma_2$ and $m_1\gamma m_2$, respectively. Call R and L the operator near-rings in a Morita context.

Examples of Morita contexts for near-rings were given in [2].

Define $[M,\Gamma] := \left\{ \sum\limits_{i=1}^{n} m_i\gamma_i | m_i \in M,\ \gamma_i \in \Gamma,\ i = 1, 2, ..., n,\ n \in N \right\}$ and $[\Gamma, M] := \left\{ \sum\limits_{j=1}^{t} \gamma_j m_j | m_j \in M,\ \gamma_j \in \Gamma,\ j = 1, 2, ..., t,\ t \in N \right\}$.

Lemma 1.1. *The sets $[M,\Gamma]$ and $[\Gamma, M]$ are subnear-rings of the operator near-rings L and R, respectively.*

Proof. It can easily be done by straightforward calculations. We only notice that in the case of composition, for example in L, we have:

$$\left(\sum_{i=1}^{n} a_i\gamma_i\right)\left(\sum_{j=1}^{m} b_j\lambda_j\right) = \sum_{i=1}^{n} a_i\omega_i, \text{ where } \omega_i = \gamma_i\left(\sum_{j=1}^{m} b_j\lambda_j\right),$$

for $i = 1, 2, ..., n$ since $\sum\limits_{j=1}^{m} b_j\lambda_j \in L$ and $\gamma_i\left(\sum\limits_{j=1}^{m} b_j\lambda_j\right) \in \Gamma$.□

All the near-rings used in the paper are right near-rings.

2. Ideals of near-rings and bigroups in Morita contexts and their relationships

Definition 2.1. Let (R, Γ, M, L) be a Morita context. A nonempty subset K of M is called an *ideal of* M (we shall denote it by $K \lhd M$), if the following conditions are satisfied:

(i) $(K, +)$ is a normal subgroup of $(M, +)$;

(ii) $kr \in K$, for all $k \in K,\ r \in R$;

(iii) $l(k + x) - lx \in K$, for all $k \in K,\ x \in M$ and $l \in L$.

The third condition is equivalent to the condition

(iii)' $l(x + k) - lx \in K$, for all $k \in K,\ x \in M$ and $l \in L$,

since K is a normal subgroup of $(M, +)$.

Definition 2.2. For $K \subseteq M$ and $j \subseteq L$, we define the sets:

$$KM^{-1} := \{l \in L | lx \in K \text{ for all } x \in M\},$$

$$J\Gamma^{-1} := \{a \in M | a\gamma \in J \text{ for all } \gamma \in \Gamma\}.$$

Similarly, for $\Delta \subseteq \Gamma$ and $I \subseteq R$, $\Delta\Gamma^{-1}$ and IM^{-1} are defined.

Recall that a subset Y of a near-ring X is an *ideal*, if $(Y, +)$ is a normal subgroup of $(X, +)$, $YX \subseteq Y$ and $x_1(y + x_2) - x_1x_2 \in Y$, for all $x_1, x_2 \in X,\ y \in Y$.

Proposition 2.3. *Let (R, Γ, M, L) be a Morita context for near-rings.*
(i) If $J \lhd L$, then $J\Gamma^{-1} \lhd M$.
(ii) If $K \lhd M$, then $KM^{-1} \lhd L$.
(iii) If $I \lhd R$, then $IM^{-1} \lhd \Gamma$.
(iv) If $\Delta \lhd \Gamma$, then $\Delta\Gamma^{-1} \lhd R$.

Proof is done by straightforward calculations.□
Similarly, by verifying the definitions, one can prove the following

Corollary 2.4. *Let (R, Γ, M, L) be a Mor near-rings. If $J \lhd L$, $N \lhd M$, $I \lhd R$ and $\Delta \lhd \Gamma$, then $J \subseteq J\Gamma^{-1}M^{-1} \lhd L$, $N \subseteq NM^{-1}\Gamma^{-1} \lhd M$, $I \subseteq IM^{-1}\Gamma^{-1} \lhd R$ and $\Delta \subseteq \Delta\Gamma^{-1}M^{-1} \lhd \Gamma$.*

Definition 2.5. M is said to have a *strong-right* (resp. *left*) *unity* if there exist elements δ (resp. ω) in Γ and e (resp. f) in M such that, for all $x \in M$, $x\delta e = x$ (resp. $f\omega x = x$).

Γ is said to have a *strong-right* (resp. *left*) *unity* if there exist elements λ (resp. α) in Γ and a (resp. b) in M such that, for all $\gamma \in \Gamma$, $\gamma a\lambda = \gamma$ (resp. $\alpha b\gamma = \gamma$).

Remark 2.6. *Assume $R = [\Gamma, M]$ and $L = [M, \Gamma]$. If M has a strong right unity (γ, e), then γe is a right identity of R.*

Indeed, for $r = \Sigma_i\gamma_i x_i \in R$,

$$r(\delta e) = (\Sigma_i\gamma_i x_i)(\delta e) = \Sigma_i\gamma_i(x_i\delta e) = \Sigma_i\gamma_i x_i = r.$$

If Γ has a strong unity (a, λ), then $a\lambda$ is a right identity of L.
Indeed, for $l = \Sigma_j y_j\omega_j \in L$,

$$l(a\lambda) = (\Sigma_j y_j\omega_j)(a\lambda) = \Sigma_j y_j(\omega_j a\lambda) = \Sigma_j y_j\omega_j = l.$$

Proposition 2.7. *Let (R, Γ, M, L) be a Morita context for near-rings. Assume $L = [M, \Gamma]$, $R = [\Gamma, M]$ and both M and Γ have strong right unities. Then for $J \lhd L$, $N \lhd M$, $I \lhd R$ and $\Delta \lhd \Gamma$, $J = J\Gamma^{-1}M^{-1}$, $N = NM^{-1}\Gamma^{-1}$, $I = IM^{-1}\Gamma^{-1}$ and $\Delta = \Delta\Gamma^{-1}M^{-1}$.*

Proof. We only have to prove the inclusions:

$$J\Gamma^{-1}M^{-1} \subseteq J \text{ and } NM^{-1}\Gamma^{-1} \subseteq N.$$

Let (f, ω) be a strong right unity of Γ. Let $l \in J\Gamma^{-1}M^{-1}$. Hence $lx\gamma \in J$, for all $x \in M$, $\gamma \in \Gamma$. Then $l = lf\omega \in J$ (taking $x = f$, $\gamma = \omega$). Thus, $J\Gamma^{-1}M^{-1} \subseteq J$.

Let (δ, e) be a strong right unity of M. If $a \in NM^{-1}\Gamma^{-1}$, then $a\gamma x \in N$, for all $\gamma \in \Gamma$, $x \in M$. Taking $\gamma = \delta$, $x = e$, we obtain

$$a = a\delta e \in N \quad \text{and} \quad NM^{-1}\Gamma^{-1} \subseteq N.$$

Other two cases are similarly proved. □

Summarizing the above results, we have:

Theorem 2.8. *Let (R, Γ, M, L) be a Morita context for near-rings, with $L = [M, \Gamma]$ and $R = [\Gamma, M]$. If both M and Γ have strong right unities, there are lattice isomorphisms between the lattice of all ideals of M, respectively of Γ,and the lattice of all ideals of L , respectively of R), given by:*

$$\begin{array}{rcl} J & \longmapsto & J\Gamma^{-1} \text{ (resp. } I \longmapsto IM^{-1}) \text{ and respectively} \\ N & \longmapsto & NM^{-1} \text{ (resp. } \Delta \longmapsto \Delta\Gamma^{-1}) \text{ , where} \\ \textit{where} & & J \rhd L \text{ and } N \rhd M \text{ (resp. } I \rhd R \text{ and } \Delta \rhd \Gamma).\square \end{array}$$

Proposition 2.9. *Let (R, Γ, M, L) be a Morita context for near-rings. Assume Γ has a strong right unity. If $I \lhd R$, then $[I, \Gamma] \lhd \Gamma$, where*

$$[I, \Gamma] := \{\Sigma_i b_i\gamma_i \in \Gamma \mid b_i \in I,\ \gamma_i \in \Gamma\}.$$

Proof. Since $0_R = 0_R\gamma \in [I, \Gamma],\ [I, \Gamma] \neq \emptyset$.

Let $\sum\limits_{i=1}^{n} b_i\gamma_i,\ \sum\limits_{j=1}^{m} c_j\omega_j \in [I, \Gamma]$, then

$$\sum_{i=1}^{n} b_i\gamma_i - \sum_{j=1}^{m} c_j\omega_j = \sum_{i=1}^{n} b_i\gamma_i + \sum_{j=m}^{1} (-c_j)\omega_j \in [I, \Gamma].$$

For all

$$\gamma \in \Gamma,\ \gamma + \Sigma_i b_i\gamma_i - \gamma = \gamma e\delta + \Sigma_i b_i(\gamma_i e\delta) - \gamma e\delta =$$

$$= (\gamma e + \Sigma_i b_i(\gamma_i e) - \gamma e)\delta \in [I, \Gamma],$$

since $\Sigma_i b_i(\gamma_i e) \in I$ and I is a normal subgroup of R. Hence, $[I, \Gamma]$ is a normal subgroup of Γ.

For all $l \in L,\ \Sigma_i b_i\gamma_i \in [I, \Gamma]$, we have $(\Sigma_i b_i\gamma_i)\, l = \Sigma_i b_i(\gamma_i l) \in [I, \Gamma]$, since $\gamma_i l \in \Gamma$.

Now, for all $r \in R, \Sigma_i b_i\gamma_i \in [I, \Gamma],\ \gamma \in \Gamma,\ r(\gamma + \Sigma_i b_i\gamma_i) - r\gamma =$

$$r(\gamma e\delta + \Sigma_i b_i\gamma_i(e\delta)) - r\gamma(e\delta) = r(\gamma e + \Sigma_i b_i(\gamma_i e)) - r(\gamma e) \in [I, \Gamma],$$

since $\Sigma_i b_i(\gamma_i e) \in I$ and $r(\gamma e + \Sigma_i b_i(\gamma_i e)) - r(\gamma e) \in I$.

Therefore $[I, \Gamma]$ is an ideal of Γ. □

Proposition 2.10. *Let (R, Γ, M, L) be a Morita context for near-rings. Assume $R = [\Gamma, M]$ and M has a strong right unity. If $\Delta \lhd \Gamma$, then $[\Delta, M] \lhd R$, where*

$$[\Delta, M] := \{\Sigma_i \delta_i x_i \in R \mid \delta_i \in \Delta,\ x_i \in M\}.$$

Proof. For all $\Sigma\delta_i x_i,\ \sum_{j=1}^{n}\delta'_j x'_j \in [\Delta, M]$,

$$\Sigma\delta_i x_i - \sum_{j=1}^{n}\delta'_j x'_j = \Sigma\delta_i x_i + \sum_{j=1}^{1}(-\delta'_j)x'_j \in [\Delta, M].$$

Take an arbitrary $r \in R$, then $r = r\omega f$ (see (2.6)). Thus,

$r + \Sigma_i\delta_i x_i - r = r\omega f + \Sigma_i\delta_i(x_i\omega f) - r\omega f = (r\omega + \Sigma_i\delta_i(x_i\omega) - r\omega)f \in [\Delta, M]$,

since $\Sigma_i\delta_i(x_i\omega) \in \Delta$ and Δ is a normal subgroup of Γ. Hence $[\Delta, M]$ is a normal subgroup of R.

For all $r \in R$ and $\Sigma_i\delta_i x_i \in [\Delta, M]$, since $x_i r \in M$,we have $(\Sigma_i\delta_i x_i)\, r = \Sigma_i\delta_i(x_i r) \in [\Delta, M]$.

For all $s \in R,\ r(s + \Sigma_i\delta_i x_i) - rs = r(s\omega f + \Sigma_i\delta_i x_i(\omega f)) - rs(\omega f) =$

$$= \{r(s\omega + \Sigma_i\delta_i(x_i\omega) - r(s\omega)\}f \in [\Delta, M],\ \text{since}\ \Sigma_i\delta_i(x\omega) \in \Delta,$$

Δ is a normal subgroup of Γ, and $r(s\omega + \Sigma_i\delta_i(x_i\omega)) - r(s\omega) \in \Delta$.

Therefore $[\Delta, M]$ is an ideal of R. $\square$

Theorem 2.11. *Let (R, Γ, M, L) be a Morita context for near-rings. Assume $R = [\Gamma, M]$ and both M and Γ have strong right unities. Then there are lattice isomorphisms between the lattice of all ideals of Γ and the lattice of all ideals of R, given, by*

$$\Delta \longmapsto [\Delta, M] \mathit{and}\ I \longmapsto [I, \Gamma], \mathit{respectively}.$$

Proof. Let $\eta : I \longmapsto [I, \Gamma]$ and $\xi : \Delta \to [\Delta, M]$. Then, for all $\Sigma_j(\Sigma_i a_{ij}\gamma_{ij})x_j \in [[I, \Gamma], M]$, we show that $\Sigma_j(\Sigma_i a_{ij}(\gamma_{ij}x_j)) \in I$. Indeed, since $\gamma_{ij}x_j \in R, a_{ij} \in I$, then $a_{ij}(\gamma_{ij}x_j) \in I$ and so

$$\Sigma_i a_{ij}(\gamma_{ij}x_j) \in I \ \text{ and }\ \Sigma_j(\Sigma_i a_{ij}(\gamma_{ij}x_j)) \in I.$$

On the other hand,

$$I = I\omega f = (I\omega)f \subseteq [[I, \Gamma], M].$$

Therefore $[[I, \Gamma], M] = I$ and this means $\xi\eta(I) = I$.

Similarly, we have $\eta\xi(\Delta) = \Delta$. $\square$

In the same manner, we prove the following two propositions.

Proposition 2.12. *Let (R, Γ, M, L) be a Morita context for near-rings. Assume M has a strong right unity. If $J \lhd L$, then $[J, M] \lhd M$, where*

$$[J, M] := \{\Sigma_i c_i x_i \mid c_i \in J,\ x_i \in M\}.\square$$

Proposition 2.13. *Let (R, Γ, M, L) be a Morita context for near-rings. Assume $L = [M, \Gamma]$ and Γ has a strong right unity. If $K \lhd M$, then $[K, \Gamma] \lhd L$, where*

$$[K, \Gamma] := \{\Sigma_i d_i \gamma_i \mid d_i \in K, \gamma_i \in \Gamma\}.\square$$

Theorem 2.14. *Let (R, Γ, M, L) be a Morita context for near-rings. Assume $L = [M, \Gamma]$ and both M and Γ have strong right unities. There are lattice isomorphisms between the lattice of all ideals of M and the lattice of all ideals of L, given by:*

$$K \longmapsto [K, \Gamma] \text{ and } J \longmapsto [J, M], \textit{respectively}.\square$$

3. Prime ideals of the near-rings and of the bigroups in a Morita context and their relationships

Let (R, Γ, M, L) be a Morita context for near-rings. We recall the definition of 3-prime ideals in the near-rings.

Definition 3.1. An ideal $J \lhd L$ is called a 3-*prime ideal*, if for any $l, l' \in L$, $lLl' \subseteq J$ implies $l \in J$ or $l' \in J$.

Let $P \lhd M$, $P \neq M$. The ideal P is called a *prime ideal in M,* if , for all $x, y \in M$, $x\Gamma y \subseteq P$ implies $x \in P$ or $y \in P$. P is a 3-*prime ideal* of M if $x\Gamma M\Gamma y \subseteq P$ implies $x \in P$ or $y \in P$, $(x, y \in M)$, where $\Gamma M\Gamma = \{\Sigma_i \gamma_i x_i \lambda_i \mid \gamma_i, \lambda_i \in \Gamma,\ x_i \in M\}$.

Proposition 3.2. *For an ideal P of M, $P \neq M$, the following statements are equivalent:*

(1) $x\Gamma y \subseteq P$, for $x, y \in M$, implies $x \in P$ or $y \in P$.

(2) $x\Gamma M\Gamma y \subseteq P$, for $x, y \in M$, implies $x \in P$ or $y \in P$.

Proof. Indeed, if (1) holds and $x\Gamma M\Gamma y \subseteq P$, $x \notin P$, then, for all $\gamma \in \Gamma$, $x\Gamma(x\gamma y) \subseteq P$. Since $x \notin P$, we have $x\gamma y \in P$; therefore $y \in P$. Conversely, if (2) holds, assume $x\Gamma y \subseteq P$. As $\Gamma M\Gamma \subseteq \Gamma$, we also have $x\Gamma M\Gamma y \subseteq P$; hence $x \in P$ or $y \in P$. Therefore (1) holds.$\square$

Theorem 3.3. *Let (R, Γ, M, L) be a Morita context for near-rings. Then the following statements hold:*

(i) If $P \lhd M$ is prime, then PM^{-1} is 3-prime.

(ii) If $J \lhd L$ is 3-prime, then $J\Gamma^{-1}$ is prime.

(iii) There is a 1-1 correspondence between the set of all prime ideals of M and the set of all 3-prime ideals of L.

Proof. *(i)* Let $l, l' \in L$ such that $lLl' \subseteq PM^{-1}$. This implies $l(x\gamma)l' \in PM^{-1}$, for all $x \in M$, $\gamma \in \Gamma$, i.e., for all $y \in M$, $l(x\gamma)l'y \in P$ (by definition of PM^{-1}). But $l(x\gamma)l'y = (lx)\gamma(l'y) \in P$, where $lx, l'y \in M$ and $\gamma \in \Gamma$ is arbitrary. Thus, $lx \in P$ or $l'y \in P$. If for all $x \in M$, $lx \in P$, then $l \in PM^{-1}$. If there exists an $x' \in M$, such that $lx' \notin P$, then for all $y \in M$ and $\gamma \in \Gamma$, $(lx')\gamma(l'y) \in P$; hence $l'y \in P$ for all $y \in M$. Therefore $l' \in PM^{-1}$ and PM^{-1} is a 3-prime ideal of L.

(ii) Let $x, y \in M$ such that $x\Gamma y \subseteq J\Gamma^{-1}$. If $x \in J\Gamma^{-1}$, then we are ready. Assume $x \notin J\Gamma^{-1}$, i.e. there is $\gamma_1 \in \Gamma$, such that $x\gamma_1 \notin J$. Taking an arbitrary $l \in L$, we have $\gamma_1 l \in \Gamma$ and $x(\gamma_1 l)y \in J\Gamma^{-1}$, i.e. for all $\gamma \in \Gamma$, $l \in L$, $(x\gamma_1)l(y\mu) \in J$. But $x\gamma_1 \notin J$, J being a 3-prime ideal, hence $y\mu \in J$ for all $\mu \in \Gamma$. Thus, $y \in J\Gamma^{-1}$.

(iii) The 1-1 correspondence is given in Theorem 2.8, taken it only for prime ideals and respectively 3-prime ideals:

$$J \longmapsto J\Gamma^{-1},\ P \longmapsto PM^{-1},$$

where $J \lhd L$ and $P \lhd M$ are 3-prime, respectively prime ideals.

It is sufficient to show the inclusions

$$J\Gamma^{-1}M^{-1} \subseteq J \text{ and } PM^{-1}\Gamma^{-1} \subseteq P,$$

because in Theorem 2.8 these inclusions were obtained by using strong unities.

Let $x \in PM^{-1}\Gamma^{-1}$. Then for all $y \in M$, $\gamma \in \Gamma$, $x\gamma y \in P$. Take $y = x$, therefore $x\gamma x \in P$ for all $\gamma \in \Gamma$. Hence $x \in P$ and $PM^{-1}\Gamma^{-1} \subseteq P$. Let $h \in J\Gamma^{-1}M^{-1}$. As $J\Gamma^{-1}M^{-1} \lhd L$, for any $l \in L$, $hl \in J\Gamma^{-1}M^{-1}$, i.e., $hlx\gamma \in J$, for all $x \in M$, $\gamma \in \Gamma$. If $J = L$, then $h \in J$. If $J \neq L$, we take $z\delta \in L \backslash J$ with $z \in M$, $\delta \in \Gamma$. Then $hlz\delta \in J$, for all $l \in L$, implying $h \in J$ since $z\delta \notin J$. Therefore, $J\Gamma^{-1}M^{-1} \subseteq J$. □

Replacing the Morita context (R, Γ, M, L) in Theorem 3.3 by the Morita context (L, M, Γ, R), we obtain:

Theorem 3.4. *Let (R, Γ, M, L) be a Morita context for near-rings. Then the following statements hold:*

$(i)'$ If $\Delta \lhd \Gamma$ is prime, then $\Delta\Gamma^{-1}$ is 3-prime.

$(ii)'$ If $I \lhd R$ is 3-prime, then IM^{-1} is prime.

$(iii)'$ There is a 1-1 correspondence between the set of all prime ideals of Γ and the set of all 3-prime ideals of R.□

We may obtain a result connecting *the prime radical of M,* namely

$$P(M) := \cap\{P | P \textit{ is prime ideal in } M\},$$

and *the 3-prime radical of the near-ring L,* namely

$$P(L) := \cap\{J \mid J \text{ is a 3-prime ideal in } L\}.$$

Corollary 3.5. *Let* (R, Γ, M, L) *be a Morita context for near-rings. Then* $P(L)\Gamma^{-1} = P(M)$ *and* $P(M)M^{-1} = P(L)$.

Proof. It is based upon the equalities $(\cap J)\Gamma^{-1} = \cap(J\Gamma^{-1})$ and $(\cap P)M^{-1} = \cap(PM^{-1})$ and Theorem 3.4.□

Remark 3.6. For $P(R)$ and $P(\Gamma)$, we have $P(R)M^{-1} = P(\Gamma)$ and $P(\Gamma)\Gamma^{-1} = P(R)$.

Definition 3.7. Let (R, Γ, M, L) be a Morita context for near-rings. Let $P \lhd L$. P is *equiprime* if there exists $x \in L\backslash P$ such that $xll_1 - xll_2 \in P$, for all $l \in L$, implies $l_1 - l_2 \in P$, where $l_1, l_2 \in L$ are arbitrary.

L is said to be *equiprime* if its zero ideal is equiprime, that is, if there exists an element $0 \neq a \in L$, such that, for $l_1, l_2 \in L$, $all_1 = all_2$,for all $l \in L$, implies $l_1 = l_2$.

Let $N \lhd M$. N is *equiprime,* if there exists $m \in M\backslash N$ such that $m\gamma x - m\gamma y \in N$ for all $\gamma \in \Gamma$ and $x, y \in M$, implies $x - y \in N$.

M is said to be *equiprime* if its zero ideal is equiprime, that is, if there exists $a \in M$, $a \neq 0$, such that for $x, y \in M$, with $a\gamma x = a\gamma y$, for all $\gamma \in \Gamma$, we have $x = y$.

Theorem 3.8. *Let* (R, Γ, M, L) *be a Morita context for near-rings.*
(i) If $P \lhd L$ *is equiprime, then* $P\Gamma^{-1} \lhd M$*, is equiprime.*
(ii) If $K \lhd M$ *is equiprime, then* $KM^{-1} \lhd L$ *is equiprime.*
(iii) There is a 1-1 correspondence between the set of all equiprime ideals of M and the set of all equiprime ideals of L, where L=[M,Γ].

Proof. *(i)* Suppose $m \in M$ and $m \notin P\Gamma^{-1}$. Then, there is $\delta \in \Gamma$ such that $m\delta \notin P$. Let, for any $\gamma \in \Gamma$, $m\gamma x - m\gamma y \in P\Gamma^{-1}$. We show that $x - y \in P\Gamma^{-1}$.

For any $\omega \in \Gamma$, $(m\gamma x - m\gamma y)\omega \in P$, i.e. $m\gamma x\omega - m\gamma y\omega \in P$.

Since γ is arbitrary, putting $\gamma = \delta l$, for any $l \in L$, we obtain $(m\delta)l(x\omega)-$ $-(m\delta)l(y\omega) \in P$.

Since $P \lhd L$ is equiprime, $x\omega - y\omega = (x - y)\omega \in P$. Hence, $x - y \in P\Gamma^{-1}$.

(ii) Suppose $a \in L$ and $a \notin KM^{-1}$. Then there exists an element $m \in M$ such that $am \notin K$.

For any $l \in L$, we assume $all_1 - all_2 \in KM^{-1}$, where $l_1, l_2 \in L$, and we shall show $l_1 - l_2 \in KM^{-1}$. For any $x \in M$, $(all_1 - all_2)x \in K$ and then $all_1x - all_2x \in K$.

For $m \in M$ and any $\gamma \in \Gamma$, $m\gamma \in L$. Therefore, putting $l = m\gamma$, we obtain $(am)\gamma(l_1x) - (am)\gamma(l_2x) \in K$.

Since $K \lhd M$ is equiprime, $l_1x - l_2x = (l_1 - l_2)x \in K$ and then $l_1 - l_2 \in KM^{-1}$.

(iii) It is sufficient to show the inclusions:

$$J\Gamma^{-1}M^{-1} \subseteq J \text{ and } KM^{-1}\Gamma^{-1} \subseteq K.$$

Let $l_1 \in J\Gamma^{-1}M^{-1}$. If $l_1 \in J$, then the statement is proved. Assume $l_1 \notin J$. For any $x \in M,\ \gamma \in \Gamma, l_1 x\gamma \in J$. For any $y \in M,\ l \in l, ly \in M$. Therefore, putting $x = ly$, we obtain $l_1(ly) \in J$,i.e., $l_1 l(y\gamma) \in J$, where $l_1 \notin J$. But if J is equiprime, then J is prime (see [1], p.3115), therefore $y\gamma \in J$. Hence $\Sigma_i y_i \gamma_i \in J$, for all $y_i \in M,\ \gamma_i \in \Gamma$. But $[M, \Gamma] = L$, hence $L \subseteq J$ and then $l_1 \in J$, a contradiction.

Let $K \lhd M$ be equiprime, we shall show that $KM^{-1}\Gamma^{-1} \subseteq K$. If $a \in KM^{-1}\Gamma^{-1}$, then, for $x \in M,\ \gamma \in \Gamma,\ a\gamma x \in K$. If $a \in K$, then the statement is proved. Assume $a \notin K$. Since if K is equiprime, then K is prime, and $a\gamma x \in K,\ a \notin K$ implies $x \in K$. As x is arbitrary, it follows $a \in K$, a contradiction.□

We define *the equiprime radical* of M by taking:

$$P_*(M) := \cap\{P \mid P \text{ is an equiprime ideal in } M\}$$

and *the equiprime radical* (see [1], p. 3117) of the near-ring L :

$$P_*(L) := \cap\{J \mid J \text{ is an equiprime ideal in} L\}.$$

Corollary 3.9. *Let (R, Γ, M, L) be a Morita context for near-rings. Assume $L = [M, \Gamma]$. Then*
$P_*(L)\Gamma^{-1} = P_*(M)$ *and* $P_*(M)M^{-1} = P_*(L)$.

Proof. It is based upon the equalities $(\cap J)\,\Gamma^{-1} = \cap(J\Gamma^{-1})$ and $(\cap P)\,M^{-1} = \cap(PM^{-1})$ and Theorem 3.8.□

If a Morita context (R, Γ, M, L) is given, considering (L, M, Γ, R) as a Morita context, we obtain the following theorem and its corollary:

Theorem 3.10. *Let (R, Γ, M, L) be a Morita context.*
(i) If the ideal Δ of Γ is equiprime, then $\Delta\Gamma^{-1}$ is equiprime.
(ii) If the ideal of R is equiprime, then IM^{-1} is equiprime.
(iii) There is a 1-1 correspondence between the set of all equiprime ideals of Γ and the set of all equiprime ideals of R, where $R = [\Gamma, M]$.□

Corollary 3.11. *For $P_*(R)$ and $P_*(\Gamma)$, we have*

$$P_*(R)M^{-1} = P_*(\Gamma) \text{ and } P_*(\Gamma)\Gamma^{-1} = P_*(R).\square$$

4. Ideals of a Morita context

Recall the definition of an ideal of a Morita context for near-rings (see [2], 4.1).

Definition 4.1. Let $A := (A_{11}, A_{12}, A_{21}, A_{22})$ be a Morita context for near-rings. Then $B := (B_{11}, B_{12}, B_{21}, B_{22})$ is an *ideal of* A, (we denote it by $B \lhd A$), if for $i, j, k \in \{1, 2\}$:

(i) B_{ij} is a normal subgroup of A_{ij};

(ii) $B_{ij}A_{jk} \subseteq B_{ik}$, where $B_{ij}A_{jk} := \{xy \mid x \in B_{ij}, y \in A_{jk}\}$;

(iii) $A_{ki} * B_{ij} \subseteq B_{kj}$, where $A_{ki} * B_{ij} = \{x(y + a) - xy \mid x \in A_{ki}, y \in A_{ij},\ a \in B_{ij}\}$.

This definition requires that each B_{ij} is not only an ideal in A_{ij}, but also it should satisfy additional conditions.

Let $(I, \Delta, K, J) \lhd (R, \Gamma, M, L)$. From the definition of an ideal, we can obtain the following mutual relationships between the ideals I and J :

$$(1) \quad \begin{cases} \Gamma * (JM) \subseteq I \\ (\Gamma * J)M \subseteq I \end{cases} \qquad \text{and } (2) \qquad \begin{cases} M * (I\Gamma) \subseteq J \\ (M * I)\Gamma \subseteq J. \end{cases}$$

Similarly, for Δ and K, we have:

$$(3) \quad \begin{cases} (M * \Delta)M \subseteq K \\ M * (\Delta M) \subseteq K \end{cases} \qquad \text{and } (4) \qquad \begin{cases} \Gamma * (K\Gamma) \subseteq \Delta \\ (\Gamma * K)\Gamma \subseteq \Delta. \end{cases}$$

If other conditions, like zerosymmetry or having strong right (left) unities, are satisfied, then we can have more strict relationships between I and J, and also between Δ and K.

For example, we assume Γ is zero-symmetric, that is

$$r0_R = 0_R,\ r0_\Gamma = 0_\Gamma,\ \gamma 0_M = 0_R,\ \gamma 0_L = 0_\Gamma,$$

$$x0_R = 0_M,\ l0_M = 0_M,\ x0_\Gamma = 0_L,\ l0_L = 0_L,$$

for all $r \in R,\ \gamma \in \Gamma,\ x \in M$ and $l \in L$.

If Γ has a right strong unity, (f, ω), that is $\gamma f \omega = \gamma$, for all $\gamma \in \Gamma$, then $\gamma(l+j)-\gamma l = (\gamma(l+j)-\gamma l)f\omega = (\gamma * j)f \in I \subseteq I\Gamma$, for all $\gamma \in \Gamma,\ j \in J,\ l \in L$. Taking $l = 0$, we obtain $\gamma j \in I\Gamma$. Hence, $x*(\gamma j) \in M*(I\Gamma) \subseteq J$, for all $x \in M$, that is, $x(\lambda + \gamma j) - x\lambda \in M * (I\Gamma) \subseteq J$, for all $\lambda \in \Gamma$.

Taking $\lambda = 0_\Gamma$, we obtain $x\gamma j \in M * (I\Gamma) \subseteq J$. Hence

$$(\Sigma_i x_i \gamma_i)\, j \in M * (I\Gamma) \subseteq J, \text{ for all } x_i \in M,\ \gamma_i \in \Gamma.$$

If $L = [M, \Gamma]$ has the left unity 1_L, then $j \in M * (I\Gamma) \subseteq J$. Thus $J \subseteq M * (I\Gamma) \subseteq J$, where $M * (I\Gamma) = J$ and then $MI\Gamma = J$.

For a subset $\Delta_{ij} \subseteq \Gamma_{ij}$, we obtained $\Delta_{ij}\Gamma_{kj}^{-1} := \{x \in \Gamma_{ik} \mid x\Gamma_{kj} \subseteq \Delta_{ij}\}$, for all $i, j, k \in \{1, 2\}$. (See [1].)

Proposition 4.2. ([2], Prop. 4.2). *Let* $\Gamma := (\Gamma_{11}, \Gamma_{12}, \Gamma_{21}, \Gamma_{22})$ *be a Morita context and* $\Delta := (\Delta_{11}, \Delta_{12}, \Delta_{21}, \Delta_{22})$ *be an ideal of* Γ. *Then, for* $i,\ j,\ k \in \{1, 2\}, \Delta_{ij}\Gamma^{-1}_{kj} \rhd \Gamma_{ij}$.□

We shall generalize the proposition as follows:

Theorem 4.3. *If* (I, Δ, K, J) *is an ideal of* $C = (R, \Gamma, M, L)$, *then*

$$\left(\Delta\Gamma^{-1}, IM^{-1}, J\Gamma^{-1}, KM^{-1}\right) \lhd (R, \Gamma, M, L).$$

Proof. Since $\Delta\Gamma^{-1} \lhd R,\ IM^{-1} \lhd \Gamma,\ J\Gamma^{-1} \lhd M,\ KM^{-1} \lhd L$, for $\Delta\Gamma^{-1}$ we shall show:

(i) $\left(\Delta\Gamma^{-1}\right)\Gamma \subseteq IM^{-1}$ and $M * (\Delta\Gamma^{-1}) \subseteq J\Gamma^{-1}$.

By the definition of $\Delta\Gamma^{-1}$, we obtain $\left(\Delta\Gamma^{-1}\right)\Gamma \subseteq \Delta$, and $\Delta M \subseteq I$, since $(I, \Delta, K, J) \lhd (R, \Gamma, M, L)$. Thus $\left(\Delta\Gamma^{-1}\right)\Gamma \subseteq \Delta \subseteq IM^{-1}$.

For any $x \in M,\ r \in R,\ a \in \Delta\Gamma^{-1}$, we have: $x(r + a) - xr \in M * (\Delta\Gamma^{-1})$, and, for any $\gamma \in \Gamma,\ (x(r + a) - xr)\gamma = x(r\gamma + a\gamma) - x(r\gamma) \in J$, since $(I, \Delta, K, J) \lhd C,\ M * \Delta \subseteq J$ and $x \in M,\ r\gamma \in \Gamma,\ a\gamma \in \Delta$.

Therefore, $M * (\Delta\Gamma^{-1}) \subseteq J\Gamma^{-1}$.

For IM^{-1}, we have to show

(ii) $(IM^{-1})M \subseteq \Delta\Gamma^{-1}$ and $M * (IM^{-1}) \subseteq KM^{-1}$.

By the definition of $IM^{-1},\ (IM^{-1})M \subseteq I$ and since $(I, \Delta, K, J) \lhd C,\ I\Gamma \subseteq \Delta$ and then $I \subseteq \Delta\Gamma^{-1}$. Therefore, $(IM^{-1})M \subseteq \Delta\Gamma^{-1}$.

For any $x(\gamma + \delta) - x\gamma \in M * (IM^{-1})$ (where $x \in M, \gamma \in \Gamma, \delta \in IM^{-1}$) and for any $y \in M$, we have $(x(\gamma + \delta) - x\gamma)y = x(\gamma y + \delta y) - x(\gamma y) \in K$, since $(I, \Delta, K, J) \lhd C,\ M * I \subseteq K$ and $x \in M,\ \gamma y \in R,\ \delta y \in I$,(because of $\Delta M \subseteq I$, due to $(I, \Delta, K, J) \lhd C$). Therefore, $M * (IM^{-1}) \subseteq KM^{-1}$.

For $J\Gamma^{-1}$ and KM^{-1}, respectively,

(iii) $(J\Gamma^{-1})\Gamma \subseteq KM^{-1}$ and $\Gamma * (J\Gamma^{-1}) \subseteq \Delta\Gamma^{-1}$,

(iv) $(KM^{-1})M \subseteq J\Gamma^{-1}$ and $\Gamma * (KM^{-1}) \subseteq IM^{-1}$

are obtained by the symmetry of a Morita context, that is, if we take an ideal (J, K, Δ, I) in the Morita context (L, M, Γ, R), then, from *(i)* and *(ii)*, it follows *(iv)* and *(iii)*, respectively.□

Definition 4.4. Let (I, Δ, K, J) be an ideal of $C = (R, \Gamma, M, L)$. If $(I, \Delta, K, J) = (\Delta\Gamma^{-1}, IM^{-1}, J\Gamma^{-1}, KM^{-1})$, then (I, Δ, K, J) is called a *upper closed ideal.*

If (I, Δ, K, J) is upper closed, then

$$(*)\ I = \Delta\Gamma^{-1},\ \Delta = IM^{-1},\ K = J\Gamma^{-1},\ \ J = KM^{-1}$$

implies

$$(**)\ \ \Delta = \Delta\Gamma^{-1}M^{-1},\ K = KM^{-1}\Gamma^{-1},\ J = J\Gamma^{-1}M^{-1} \quad .$$

In general, since the relations:

$$I \subseteq \Delta\Gamma^{-1} \subseteq IM^{-1}\Gamma^{-1}, \quad \Delta \subseteq IM^{-1} \subseteq \Delta\Gamma^{-1}M^{-1},$$

$$K \subseteq J\Gamma^{-1} \subseteq KM^{-1}\Gamma^{-1}, \quad J \subseteq KM^{-1} \subseteq J\Gamma^{-1}M^{-1},$$

hold, if $(**)$ holds, then (I, Δ, K, J) is upper closed. Therefore, $(*)$ and $(**)$ are equivalent.

As we have seen in § 3, an ideal, when M and Γ have strong right unities, a prime ideal and an equiprime ideal are upper closed.

If a Morita context $C = (R, \Gamma, M, L)$ is given, we obtain a new Morita context $C^0 := (L, M, \Gamma, R)$, immediately.

Now we have:

Theorem 4.5. *Let $C = (R, \Gamma, M, L)$ be a Morita context for near-rings and (I, Δ, K, J) be an ideal of C. Then, we obtain a Morita context for near-rings: $(R/I, \Gamma/\Delta, M/K, L/J)$.*

Proof. It is obvious that $\bar{R} = R/I$ and $\bar{L} = L/J$ are near-rings, while $\bar{\Gamma} = \Gamma/\Delta$ and $\overline{M} = M/K$ are groups. We shall show only that $\overline{M}$ is an $\bar{L}$-$\bar{R}$-bigroup. For $\bar{l} \in \bar{L}$, $\bar{x} \in \overline{M}$, define $\bar{l}\bar{x} := \overline{lx}$, and it is well-defined. Indeed, let $l' = l + j$, $x' = x + k$, where $j \in J$, $k \in K$. Then

$$l'x' = (l + j)(x + k) = l(x + k) + j(x + k) =$$

$$= (l(x + k) - lx) + (lx + j(x + k) - lx) + lx = k' + lx,$$

where $k' \in K$, since $l(x + k) - lx \in L * K \subseteq K$ and $lx + j(x + k) - lx \in K$.

Therefore, $\overline{l'x'} = \overline{lx}$.

It is easily to verify that M is a left L-group.

Similarly, by defining: $\bar{x} \cdot \bar{r} := \overline{xr}$, for any $\bar{x} \in \overline{M}$, $r \in \overline{R}$, $\overline{M}$ becomes a right $\overline{R}$-group (easy verification!).

Now $\overline{M}$ is an $\bar{L}$-$\bar{R}$-bigroup and $\overline{\Gamma}$ is an $\bar{R}$-$\bar{L}$-bigroup; using the function: $\overline{\Gamma} \times \overline{M} \to \overline{R}$, $(\bar{\gamma}, \bar{x}) \to \bar{\gamma}\bar{x}$, defined by $\bar{\gamma}\bar{x} := \overline{\gamma x}$, and similarly, $\bar{x}\bar{\gamma} := \overline{x\gamma}$, for any $\bar{x} \in \overline{M}$, $\bar{\gamma} \in \overline{\Gamma}$, we see that $(\overline{R}, \overline{\Gamma}, \overline{M}, \overline{L})$ is a Morita context.

If we assume $[\Gamma, M]{=}R$ and $[M, \Gamma] = L$, then $[\overline{\Gamma}, \overline{M}]{=}\overline{R}$ and $[\overline{M}, \overline{\Gamma}] = \overline{L}$ since $r = \overline{\Sigma_i \gamma_i x_i} = \Sigma_i \overline{\gamma_i x_i} = \Sigma_i \bar{\gamma}_i \bar{x}_i$ and $\bar{l} = \overline{\Sigma_j y_j \omega_j} = \Sigma_j \overline{y_j \omega_j} = \Sigma_j \bar{y}_j \bar{\omega}_j$.$\square$

5. An example of Morita context for near-rings

Let R be a unitary near-ring, and take 1_R as the identity of R. Fixing $r \in R$, let $f^r : R \to R$ be a mapping defined by $f^r(x) = rx$, for any $x \in R$; let π_j be a projection $\begin{pmatrix} R \\ R \end{pmatrix} \to R$, where $j \in \{1,2\}$, and σ_i be an injection $R \to \begin{pmatrix} R \\ R \end{pmatrix}$, where $i \in \{1,2\}$. Since R has identity, we have:

$$R \cong f^R := \{f^r \in Map(R,R) \mid r \in R\}.$$

Let L be the sub-near-ring of $Map\left(\begin{bmatrix} R \\ R \end{bmatrix}, \begin{bmatrix} R \\ R \end{bmatrix}\right)$ generated by $\{\sigma_i \circ f^r \circ \pi_j \mid r \in R,\ i,j \in \{1,2\}\}$.

Then $L = M_2(R)$, which is the matrix near-ring over R (see [3]).

Let P be the left R-right L-bisubgroup of $Map\left(\begin{bmatrix} R \\ R \end{bmatrix}, R\right)$, generated by $\{f^r \circ \pi_j \mid r \in R,\ j \in \{1,2\}\}$.

Let Q be the left L-right R-bisubgroup of $Map\left(\begin{bmatrix} R \\ R \end{bmatrix}\right)$, generated by $\{\sigma_i \circ f^r \mid r \in R,\ i \in \{1,2\}\}$.

Example 5.1. $\Gamma_o = (R, P, Q, L)$ is a Morita context for near-rings.

Proof. By definitions of P and Q, it is easy to see that
(i) P is an R-L-bigroup and *(ii)* Q is an L-R-bigroup.
To show *(iii)*, define $P \times Q \to R$, by $(p,q) \to p \circ q$, mapping composition, and $p := f^r \circ \pi_1 + f^s \circ \pi_2$, $q := \sigma_1 \circ f^u + \sigma_2 \circ f^v$.
Then $p \circ q(x) = (ru + sv)x$, where $ru + sv \in R$.
It is easy to verify other relations like: $(p + p') \circ q = p \circ q + p' \circ q$.
To show *(iv)*, define $Q \times P \to L$, by $(q,p) \to q \circ p$, where $q \circ p \begin{bmatrix} x \\ y \end{bmatrix} := q\left(p \begin{bmatrix} x \\ y \end{bmatrix}\right)$, for all $x, y \in R$ and $p := f^r \circ \pi_1 + f^s \circ \pi_2$, $q := \sigma_1 \circ f^u + \sigma_2 \circ f^v$. Then $q \circ p \begin{bmatrix} x \\ y \end{bmatrix} = \sigma_1 \circ f^u \circ \pi_1 \circ (\sigma_1 \circ f^r \circ \pi_1$ $+ \sigma_1 \circ f^s \circ \pi_2 \begin{bmatrix} x \\ y \end{bmatrix} + \sigma_2 \circ f^v \circ \pi_1 \circ (\sigma_1 \circ f^r \circ \pi_1 + \sigma_1 \circ f^s \circ \pi_2) \begin{bmatrix} x \\ y \end{bmatrix}$,
where $\sigma_1 \circ f^u \circ \pi_1 \circ (\sigma_1 \circ f^r \circ \pi_1 + \sigma_1 \circ f^s \circ \pi_2) +$

$$+\sigma_1 \circ f^v \circ \pi_1 \circ (\sigma_1 \circ f^r \circ \pi_1 + \sigma_1 \circ f^s \circ \pi_2) \in L.$$

The other relations are easily verified.

(v) Since $p \circ q,\ q \circ p$ are mapping compositions, we obtain $p' \circ (q \circ p'') = (p' \circ q) \circ p''$ and $(q' \circ p) \circ q'' = q' \circ (p \circ q'')$, for all $p, p', p'' \in P,\ q, q', q'' \in Q$.

Remarks 5.2. (1) Taking $\begin{pmatrix} R \\ \vdots \\ R \end{pmatrix}$ instead of $\begin{pmatrix} R \\ R \end{pmatrix}$, we can obtain a Morita context $\Gamma_o = (R, P, Q, L)$, where $L = M_n(R)$.

(2) One of the important roles of Morita contexts is to investigate the close relations between R and L (see § 6). To aim this we must construct a special ideal. For $\Gamma_0 = (R, P, Q, L)$, where $L = M_2(R)$, let $A < R$ and define $A_2 := \left\{ x \in L | x \begin{bmatrix} R \\ R \end{bmatrix} \subseteq \begin{bmatrix} A \\ A \end{bmatrix} \right\}$. Then A_2 is an ideal of L, generated by $\{\sigma_1 \circ f^a \circ \pi_j \mid a \in A,\ i, j \in \{1, 2\}\}$, since $\sigma_i \circ f^r \circ \pi_j \begin{pmatrix} 1_R \\ 1_R \end{pmatrix} = \begin{bmatrix} r \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 0 \\ r \end{bmatrix} \in \begin{bmatrix} A \\ A \end{bmatrix}$. Therefore, $AQ^{-1} \lhd P$ and $A_2 P^{-1} \lhd Q$.

Now we have

Proposition 5.3. *For the Morita context* $\Gamma_o = (R, P, Q, L)$, *where* $L = M_2(R)$, $\Delta := (A, AQ^{-1}, A_2 P^{-1}, A_2)$ *is an ideal of* Γ_o.

Proof. (1) Since $AQ^{-1} \lhd P$, we have $R*(AQ^{-1}) \subseteq AQ^{-1}$ and $(AQ^{-1})L \subseteq AQ^{-1}$. By the definition of $AQ^{-1}, (AQ^{-1})Q \subseteq A$. We shall show that $Q * (AQ^{-1}) \subseteq A_2$. For any $q \in Q, p \in P, \delta \in AQ^{-1}$, $\begin{bmatrix} x \\ y \end{bmatrix} \in \begin{bmatrix} R \\ R \end{bmatrix}$,

$$(q(p+\delta) - qp) \begin{bmatrix} x \\ y \end{bmatrix} = q\left(p \begin{bmatrix} x \\ y \end{bmatrix} + \delta \begin{bmatrix} x \\ y \end{bmatrix}\right) - qp \begin{bmatrix} x \\ y \end{bmatrix} = q(r+a) - qr,$$

where $p \begin{bmatrix} x \\ y \end{bmatrix} = r,\ \delta \begin{bmatrix} x \\ y \end{bmatrix} = a$.

We prove that $\delta \begin{bmatrix} x \\ y \end{bmatrix} \in A$.

Let $\delta := r_1\pi_1 + r_2\pi_2 \in AQ^{-1}$. Then $(r_1\pi_1 + r_2\pi_2)\pi_1 = r_1\pi_1\sigma_1 = r_1 \in A$. Thus $r_1 = a_1 \in a$. $(r_1\pi_1 + r_2\pi_2)\sigma_2 = r_2\pi_2\sigma_2 = r_2 \in A$. Thus $r_2 = a_2 \in A$. Thus, $\delta = a_1 p_1 + a_2 p_2$ and so $\delta \begin{bmatrix} x \\ y \end{bmatrix} = a_1 p_1 \begin{bmatrix} x \\ y \end{bmatrix} + a_2 p_2 \begin{bmatrix} x \\ y \end{bmatrix} = a_1 x + a_2 y \in A$, since $A \lhd R$.

Taking $q(u) = \begin{bmatrix} tu \\ su \end{bmatrix}$, where $t, s \in R$, then

$$q(r+a) - qr = \begin{bmatrix} t(r+a) - tr \\ s(r+a) - sr \end{bmatrix} \in \begin{bmatrix} A \\ A \end{bmatrix}.$$

Therefore, $Q * (AQ^{-1}) \subseteq A_2$.

(2) Since $A_2P^{-1} \lhd Q$, we have $\{(A_2P^{-1})R \subseteq A_2p^{-1},\ L*(A_2P^{-1}) \subseteq$ $\subseteq A_2P^{-1},\ (A_2P^{-1})P \subseteq A_2$, by the definition of A_2P^{-1}. We shall show $P*(A_2P^{-1}) \subseteq A$.

For any $p \in P,\ q \in Q,\ \delta \in A_2P^{-1},\ x \in R,\ (p(q+\delta)-pq)x =$ $= p(qx+\delta x) - p(qx)$.

Let $\delta = \sigma_1 r_1 + \sigma_2 r_2$. Since $(\Sigma_{ij}\sigma_i \cdot r_{ij} \cdot \pi_j)(\sigma_1 0 + \sigma_2 0) = \sigma_1 0 + \sigma_2 0 = 0$, for $l(q+a) - lq \in L*(A_2P^{-1})$, where $l \in L, q \in Q,\ a \in A_2P^{-1}$, putting $q = 0$, we have $la \in L*(A_2P^{-1})$. Thus $L(A_2P^{-1}) \subseteq L*(A_2P^{-1})(\subseteq A_2P^{-1})$ and then $L(A_2P^{-1}) \subseteq A_2P^{-1}$. Thus

$$\begin{bmatrix} 1_R & 0 \\ 0 & 0 \end{bmatrix} \delta = \begin{bmatrix} 1_R & 0 \\ 0 & 0 \end{bmatrix} (\sigma_1 r_1 + \sigma_2 r_2) = \sigma_1 r_1 \in A_2P^{-1},$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1_R \end{bmatrix} \delta = \sigma_2 r_2 \in A_2P^{-1}.$$

Then $\sigma_1 r_1 p_1 \in A_2,\ \sigma_1 r_2 \pi_2 \in A_2$,

$$\delta x = (\sigma_1 r_1 + \sigma_2 r_2)\, x = \sigma_1 r_1 x + \sigma_2 r_2 x = \begin{bmatrix} r_1 x \\ r_2 x \end{bmatrix} = (\sigma_1 r_1)\, \pi_1 \begin{bmatrix} x \\ x \end{bmatrix} +$$

$$+ (\sigma_2 r_2)\, \pi_2 \begin{bmatrix} x \\ x \end{bmatrix} \in \begin{bmatrix} A \\ A \end{bmatrix},\ \text{since } \sigma_i r_i \pi_i \in A_2.$$

Let $\delta x = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$, where $a_1, a_2 \in A$, and $qx = \begin{bmatrix} t \\ s \end{bmatrix}$. Then

$$p(qx+\delta x) - p(qx) = p\left(\begin{bmatrix} t \\ s \end{bmatrix} + \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) - p\begin{bmatrix} t \\ s \end{bmatrix}.$$

Putting $p = k\pi_1 + h\pi_2$,

$$= (k\pi_1 + h\pi_2)\begin{bmatrix} t+a_1 \\ s+a_2 \end{bmatrix} - (k\pi_1 + h\pi_2)\begin{bmatrix} t \\ s \end{bmatrix} =$$

$$= k\pi_1 \begin{bmatrix} t+a_1 \\ s+a_2 \end{bmatrix} + h\pi_2 \begin{bmatrix} t+a_1 \\ s+a_2 \end{bmatrix} - ((k\pi_1)\begin{bmatrix} t \\ s \end{bmatrix} + h\pi_2 \begin{bmatrix} t \\ s \end{bmatrix}) =$$

$$= k(t+a_1) + h(s+a_2) - (kt+hs) = k(t+a_1) + h(s+a_2) - hs - kt =$$

$$= (k(t+a_1) - kt) + (kt + (h(s+a_2) - hs) - kt) \in A,$$

since $a_1, a_2 \in A \lhd R$.

(3) Since $A \lhd R$, we have $AR \subseteq A$ and $R*A \subseteq A$. Since $(AP)Q\ \ AR\ \ A$, we have $AP \subseteq AQ^{-1}$. We shall show that $Q*A \subseteq A_2P^{-1}$. To end, we show

that $(Q*A)P \subseteq A_2$. For any $q \in Q,\ r \in R,\ a \in A, p \in P,$ $\begin{bmatrix} x \\ y \end{bmatrix} \in \begin{bmatrix} R \\ R \end{bmatrix}$,we have

$$(q(r+a)-qr)p\begin{bmatrix} x \\ y \end{bmatrix} = q(rs+as)-q(rs), \text{ where } s = p\begin{bmatrix} x \\ y \end{bmatrix}.$$

Putting $q(u) = \begin{bmatrix} hu \\ ku \end{bmatrix}$, where $h, k, u \in R$,
we get the membership $\begin{bmatrix} h(rs+as)-h(rs) \\ k(rs+as)-k(rs) \end{bmatrix} \in \begin{bmatrix} A \\ A \end{bmatrix}$. □

(4) For A_2, since $A_2 < L$, we have $A_2L \subseteq A_2, L * A_2 \subseteq A_2$.
Since $(A_2Q)P \subseteq A_2L \subseteq A_2$, we obtain $A_2Q \subseteq A_2P^{-1}$.

Now, we shall show $P * A_2 \subseteq AQ^{-1}$, that is, $(P * A_2)Q \subseteq A$.

For any $p \in P,\ l \in L,\ a \in A_2,\ q \in Q,\ x \in R,$

$$(p(l+a)-pl)q(x) = p\left(l\begin{bmatrix} t \\ s \end{bmatrix} + a\begin{bmatrix} t \\ s \end{bmatrix}\right) - pl\begin{bmatrix} t \\ s \end{bmatrix},$$

where $q(x) = \begin{bmatrix} t \\ s \end{bmatrix}$.

Putting $l\begin{bmatrix} t \\ s \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix}$, $a\begin{bmatrix} t \\ s \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$, we have

$$(p(l+a)-pl)q(x) \text{ equal to } p\left(\begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) - p\begin{bmatrix} u \\ v \end{bmatrix} \text{ in } A,$$

as we have shown that $P * (A_2P^{-1}) \subseteq A$ in (2). □

Remarks 5.4. For the Morita context $\Gamma = (R, P, Q, L)$, where $L = M_n(R)$, we have an ideal $(A, AQ^{-1}, A_nP^{-1}, A_n)$, where $A \lhd R,\ R \ni 1_R$ and
$A_n := \left\{ x \in M_n(R) \mid x \begin{bmatrix} R \\ \vdots \\ R \end{bmatrix} \subseteq \begin{bmatrix} A \\ \vdots \\ A \end{bmatrix} \right\}.$

6. A representation of a Morita context for near-rings

Let $C = (R, \Gamma, M, L)$ be a Morita context for near-rings. Using the near-ring R and the right R-group M_R from C, we can construct a Morita context for near- rings: (R, M^*, M, S) [see [2], Example 3], where $M^* = Map_R(M, R) := \{f \mid f : M \to R$ is a function such that $f(xr) = f(x)r$, for all $x \in M$ and $r \in R\}$, and $S = Map_R(M, M) :=$

$:= \{s \mid s : M \to M$ is a function such that $s(xr) = s(x)r$, for all $x \in M$ and $r \in R\}$.

Theorem 6.1. *Suppose that (R, Γ, M, L) is a Morita context for near-rings. Assume $\gamma M = 0$ implies $\gamma = 0$ and M has a strong left unity. Then*
(1) $\Gamma \cong M^*$ as bigroups;
(2) $L \cong Map_R(M, M)$ as near-rings.
Assume $x\Gamma = 0$ implies $x = 0$ and Γ has a strong left unity. Then
(3) $M \cong \Gamma^*$ as bigroups;
(4) $R \cong Map_R(\Gamma, \Gamma)$ as near-rings.

Proof. (1) Consider a map $\Gamma \to M^*$, $\gamma \to \bar{\gamma}$, where, for $\forall m \in M$, $\bar{\gamma}(m) := \gamma m \in R$. Since $\bar{\gamma}(mr) = \gamma(mr) = \bar{\gamma}(m)r$, then $\bar{\gamma} \in M^*$. For any $m \in M$, $\overline{\gamma + \gamma'}(m) = (\gamma + \gamma')m = \gamma m + \gamma' m = \bar{\gamma}(m) + \bar{\gamma}'(m)(\bar{\gamma} + \bar{\gamma}')(m)$.

Therefore $\overline{\gamma + \gamma'} = \bar{\gamma} + \bar{\gamma}'$ and similarly $\overline{r\gamma} = r\bar{\gamma}$, $\overline{\gamma l} = \bar{\gamma} l$.

Therefore „$-$" is a bigroup homomorphism. To see it is injective, let $\gamma = \omega$, then for any $m \in M$, $\bar{\gamma}(m) = \overline{\omega}(m)$, $\gamma m = \omega m$, $(\gamma + (-\omega))m = 0$.

By the assumption "$\gamma M = 0 \Rightarrow \gamma = 0$", we obtain $\gamma + (-\omega) = 0$, that is $\gamma = \omega$.

Let (e, δ) be a left unity of M. For any $f \in M^*$, $f(e) \in R$ and then $f(e)\delta \in R\delta \subseteq \Gamma$.

For any $x \in M$, $\overline{f(e)\delta}(x) = (f(e)\delta)x = f(e)(\delta x) = f(e\delta x) = f(x)$.

Therefore, $\overline{f(e)\delta} = f$.

Therefore, „$-$" is surjective and therefore it is a bigroup isomorphism.

(2) Now consider the mapping „$-$" : $L \to Map_R(M, M)$, $l \longmapsto \bar{l}$, where for any $m \in M$, $\bar{l}(m) := lm \in M$. By straightforward calculations, we prove that „$-$" is an isomorphism of near-rings. From the Morita context $C = (R, \Gamma, M, L)$, we obtain a Morita context $C' = (L, M, \Gamma, R)$.

From C', we have a Morita context (L, Γ^*, Γ, U), where $\Gamma^* = Map_L(\Gamma, L)$ and $U = Map_L(\Gamma, \Gamma)$.

Under the assumptions „$x\Gamma = 0 \Rightarrow x = 0$" and the existence of a strong left unity (ω, f) of Γ, i.e., $\omega f \gamma = \gamma$ for all $\gamma \in \Gamma$, we obtain, similarly, (3) and (4).$\Box$

Corollary 6.2. *Let (R, Γ, M, L) be a Morita context for near-rings. Assume $\gamma M = 0$ implies $\gamma = 0$ and $x\Gamma = 0$ implies $x = 0$, both M and Γ having strong left unities; then $M \cong (M^*)^*$ and $\Gamma \cong (\Gamma^*)^*$ as bigroups; $R \cong Map_{Map_R(M,M)}(M^*, M^*)$ and*
$L \cong Map_{Map_R(\Gamma,\Gamma)}(\Gamma^*, \Gamma^*)$ as near-rings.$\Box$

Remark 6.3. *If Γ has a strong right unity (f, ω), then $\gamma M = 0$ implies $\gamma = 0$, since $\gamma M = 0$, and then $\gamma f = 0f = 0$, whence $\gamma = \gamma f \omega = 0\omega = 0$.*

Similarly, if M has a strong right unity (ω, f), then $x\Gamma = 0$ implies $x = 0$.

Thus, both M and Γ have strong unities, then Theorem 6.1 and Corollary 6.2 hold.

7. Equivalence between a category of right R-groups and a category of right L-groups for a Morita context (R, Γ, M, L)

Let (R, Γ, M, L) be a Morita context. Let $[M, R] := \{\Sigma_i m_i r_i \in M \mid m_i \in M, r_i \in R\}$. Then, for any $x \in M,\ x\omega f \in [M, R]$ and so $[M, R] = M$, where (ω, f) is a right strong unity of M.

If $xR = 0$, then $x = x\omega f = 0$, since $\omega f \in R$.

In the following, $\mathcal{G}(R)$ denotes the category of right R-groups where the morphisms are R-group homomorphisms, that is, let G_R, G'_R be right R-groups, a map $f : G \to G'$ is said to be an R-group homomorphism if $f(g_1 + g_2) = f(g_1) + f(g_2)$ and $f(gr) = f(g)r$, for all $g_1, g_2, g \in G$ and $r \in R$.

Similarly, $\mathcal{G}(L)$ denotes the category of right L-groups over L where morphisms are L-group homomorphisms.

The following theorem gives the equivalence of the categories $\mathcal{G}(R)$ and $\mathcal{G}(L)$.

Theorem 7.1. *Let (R, Γ, M, L) be a Morita context for near-rings. Assume $R = [\Gamma, M]$, $L = [M, \Gamma]$, M has a strong right unity and Γ has a strong right unity. Then the categories of right R-groups and right L-groups are equivalent.*

Proof. Let $G \in 0b\mathcal{G}(R)$. Let A be a free additive group generated by the set of ordered pairs (g, γ), where $g \in G,\ \gamma \in \Gamma$, and let B be the subgroup of elements $\Sigma_i m_i(g_i, \gamma_i) \in A$, where m_i are integers such that $\Sigma_i m_i g_i(\gamma_i x) = 0$, i.e. $B := \{\Sigma_i m_i(g_i\gamma_i) \in A \mid \Sigma_i m_i g_i(\gamma_i x) = 0, \text{ for all } x \in M\}$. Then B is a normal subgroup of A. Indeed, for any $a \in A$ and $b \in B,\ x \in M$,

$$(a + b - a)x = ax + bx - ax = ax + 0 - ax = 0,$$

where for $a = \Sigma_i m_i(g_i\gamma_i)$ and $x \in M$, we define $ax := \Sigma_i m_i g_i(\gamma_i x)$.

Therefore, we can make a factor group A/B and denote it by $[G, \Gamma]$. Denote by $[g, \gamma]$ the coset $(g, \gamma) + B$. We have $[g_1, \gamma] + [g_2, \gamma] = [g_1 + g_2, \gamma]$ and $[gr, \gamma] = [g, r\gamma]$. Each element in $[G, \Gamma]$ can be expressed as a finite sum $\Sigma_i[g_i, \gamma_i]$.

$[G, \Gamma]$ is a right L-group with respect to the external operation

$$(\sigma_i[g_i, \gamma_i])\, l = \Sigma_i[g_i(\gamma_i lf), \omega] = [\Sigma_i g_i(\gamma_i lf), \omega] \in [G, \Gamma],$$

since $l = lf\omega,\ lf \in M$ and $\gamma_i(lf) \in R$.

An R-group homomorphism $f : G_R \to H_R$ determines an L-group homomorphism $\bar{f} : [G,\Gamma] \to [H,\Gamma]$ by $f\,(\Sigma_i[g_i,\gamma_i]) := \Sigma_i[f(g_i),\gamma_i]$, where $g_i \in G,\ \gamma_i \in \Gamma$.

Since $\Sigma_i[g_i,\gamma_i] = \Sigma_j[g'_j,\gamma'_j]$ implies, for any $x \in N$, the equality: $\Sigma_i g_i(\gamma_i x) = \Sigma_j g'_j(\gamma'_j x)$, we can show that $\bar{f}$ is well-defined.

Now, let us verify that $[G,\Gamma] \in 0b\mathcal{G}(L)$.
For any $\Sigma_i[g_i,\gamma_i],\ \Sigma_i[g'_i,\gamma'_i] \in [G,\Gamma]$ and $l, l' \in L$,

$$(\Sigma_i[g_i,\gamma_i] + \Sigma_i[g'_i,\gamma'_i])\, l = [\Sigma_i g_i(\gamma_i l f) + \Sigma_j g'_j(\gamma'_j l f),\omega] =$$

$$= [\Sigma_i g_i(\gamma_i l f),\omega] + [\Sigma_i g'_i(\gamma'_i l f),\omega] = \Sigma_i[g_i,\gamma_i]l + \Sigma_j[g'_j,\gamma'_j]l$$

and

$$\Sigma_i[g_i,\gamma_i](ll') = \Sigma_i g_i \gamma_i (ll')f,\omega] = [\Sigma_i g_i(\gamma_i l f \omega l')f,\omega] =$$

$$= [\Sigma_i g_i(\gamma_i l f),\omega]l' = (\Sigma_i[g_i,\gamma_i]l)\, l'.$$

Similarly, for $U \in 0b\mathcal{G}(L)$, we can define a right R-group $[U,M]$ and show $[U,M] \in 0b\mathcal{G}(R)$.

The above defined $\bar{f}$ is also an L-group homomorphism.

Similarly, an L-group homomorphism $h : U_L \to V_L$ determines an R-group homomorphism $\bar{h} : [U,M]_R \to [V,M]_R$ by $\bar{h}\,(\Sigma_j[u_j,x_j]) := \Sigma_j[h(u_j),x_j]$.

Let f_1 and f_2 be R-group maps $f_1 : A \to B,\ f_2 : B \to C$.

Let $\bar{f}_1$ and $\bar{f}_2$ be the L-group homomorphisms determined by f_1 and f_2, respectively.

Then, $f_2 \circ f_1 : A \to C$ determines an $L-$group homomorphism $p : [A,\Gamma] \to [C,\Gamma]$, where $p = \bar{f}_2 \circ \bar{f}_1$. Indeed, for any $\Sigma_i[a_i,\gamma_i] \in [A,\Gamma]$, we have

$$p\,(\Sigma_i[a_i,\gamma_i]) = \Sigma_i[f_2 \circ f_1(a_i),\gamma_i] = \Sigma_i[f_2(f_1(a_i)),\gamma_i] =$$

$$= \bar{f}_2\,(\Sigma_i[f_1(a_i),\gamma_i]) = \bar{f}_2 \circ \bar{f}_1\,(\Sigma_i[a_i,\gamma_i])\,.$$

Clearly, $1_A : A \to A$ determines $1_{[A,\Gamma]} : [A,\Gamma] \to [A,\Gamma]$.

Thus, we have functors:

$$F : \mathcal{G}(R) \to \mathcal{G}(L) \text{ and } H : \mathcal{G}(L) \to \mathcal{G}(R),$$

where for $A \in 0b\mathcal{G}(R),\ F(A) = [A,\Gamma]$ and for $U \in 0b\mathcal{G}(L),\ H(U) = [U,M],\ HF(A) = H\,([A,\Gamma]) = [[A,\Gamma],M]$ and $FH(U) = F\,([U,M]) = [[U,M],\Gamma]$.

Define $\eta_A : A = AR = A[\Gamma,M] \to [[A,\Gamma],M]$ by $a = \Sigma a_i r_i \to [\Sigma_i[a_i,r_i\delta],e]$.

Assume $[\Sigma_i[a_i,r_i\delta],e] = 0$.Then $0 = ([\Sigma_i[a_i,r_i\delta])\,e$, by definition of $[[A,\Gamma],M]$ and for any $\gamma \in \Gamma,\ 0 = \Sigma_i[a_i(r_i\delta(e\gamma f)),\omega]$, by definition of $[A,\Gamma] \times L$.

We get further:

$0 = \Sigma_i\,[a_i(r_i\gamma f),\omega]$, by Definition 1.1 *(v),(iii)*,

$0 = \Sigma_i\,[a_i r_i(\gamma f),\omega]$, since M is a right R-group,

$0 = \Sigma_i \left[a_i r_i, (\gamma f), \omega\right]$, since $[ar, \omega] = [a, r\overline{\omega}]$,
$0 = \left[\Sigma_i a_i r_i, \gamma(f\omega)\right]$, by Definition 1.1*(iii)*,
$0 = \left[\Sigma_i a_i r_i, \gamma\right]$, by using the fact that $f\omega$ is a strong unity of Γ.

Putting $\gamma=\delta$, we obtain $[\Sigma_i a_i r_i, \delta] = 0$. Then, for any $x \in M, \Sigma_i a_i(r_i\delta x) = 0$,and by taking $x = e, \Sigma_i a_i(r_i\delta e) = \Sigma_i a_i x_i = 0$.

Therefore, η_A is an injection.

For any $b = \Sigma_j \left[\Sigma_i \left[a_{ij}, \gamma_{ij}\right], x_j\right] \in \left[\left[A, \Gamma\right], M\right]$, there exists an element $a = \Sigma_j \left(\Sigma_i a_{ij} \left(\gamma_{ij} x_j\right)\right) \in AR = A$,such that $\eta_A(a) = b$.

Therefore, η_A is a bijection.

To see η_A is an R-group homomorphism, for any $a = \Sigma_i a_i r_i,\ b = \Sigma_i b_j s_j \in A,\ r \in R$, we verify the conditions:

$$\eta_A(a+b) = \left[\Sigma_i \left[a_i, r_i\delta\right] + \Sigma_i \left[b_j, s_j\delta\right], e\right] =$$

$$= \left[\Sigma \left[a_i, r_i\delta\right], e\right] + \left[\Sigma_j \left[b_j, s_j\delta\right], e\right] = \eta_A(a) + \eta_A(b).$$

$$\eta_A(ar) = \left[\Sigma_i \left[a_i, r_i\delta er\right], e\right] = \left[\Sigma_i \left[a_i, r_i\delta e\right] r\delta, e\right] = \left[\Sigma_i \left[a_i, r_i\delta\right], e\right] r = \eta_A(a)r.$$

For an R-group homomorphism $f : A_R \to B_R$ and for $a = \Sigma_i a_i r_i = \Sigma_i a_i(r_i\delta)e \in A$, we have

$$HF(f)\eta_A(a) = HF(f)\eta_A\left(\Sigma_i a_i \left(r_i\delta\right) e\right) = HF(f)\Sigma_i \left[\left[a_i, r_i\delta\right], e\right] =$$

$$= \Sigma_i[F(f)([a_i, r_i\delta], e]) = \Sigma_i[f(a_i), r_i\delta], e] = \eta_B f(a).$$

Therefore, we have the following commutative diagram:

$$\begin{tikzcd}
\mathcal{G}(R) \arrow[r, "F"] \arrow[dr, "1_{\mathcal{G}(R)}"'] & \mathcal{G}(L) \arrow[d, "H"] \\
 & \mathcal{G}(R)
\end{tikzcd}$$

Thus, $HF = 1_{\mathcal{G}(R)}$. Similarly, we obtain $FH = 1_{\mathcal{G}(L)}$.

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