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ANTIPLANE FRACTURE ANALYSIS OF A CRACK IN A MONOCLINIC COMPOSITE

Eduard-Marius Craciun

To Professor Silviu Sburlan, at his 60's anniversary

Abstract

The behavior of the incremental fields near the crack tips of a prestressed monoclinic material in antiplane state is presented. Incremental displacement and stress fields are obtained using complex potentials. Using Griffith's energy criterion are given the critical values of the incremental stress which produces crack propagation. The occurance of the resonance phenomenon for monoclinic and for orthotropic composites is studied in the paper.

1 Introduction

We consider a prestressed monoclinic linear elastic material representing a fiber reinforced composite. We assume that the loss of internal stability for the prestressed equilibrium state cannot take place. We suppose that the monoclinic material is unbounded and contains a crack of length $2a$, parallel to the reinforcing fibers and to the initial applied stresses. We assume that the crack faces are acted by antiplane tearing incremental stresses.

Using the theories of the Riemann-Hilbert function and Cauchy's integral we give the solution to our mathematical problem. Our first aim is to determine the behavior of incremental fields of our monoclinic material containing a crack. The obtained results are presented in the Section 2 and 3. In Section 4 are presented the asymptotical behavior of the incremental fields necessary to determine the values of the critical tearing stresses which are producing the crack propagation; and this represents our second aim. In Section 5, our third and last aim represents the study of the resonance phenomenon in monoclinic composites containing a crack and particularly in orthotropic materials.

Key Words: Antiplane crack, prestressed composite, resonance.
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2 Incremental fields equations for a monoclinic material.

We suppose that the whole space is occupied by a monoclinic material having the initial deformed equilibrium configuration *homogenous* and locally stable. Let $\overset{\circ}{\sigma}$ be initial applied stress acting in the direction of reinforcing fibers of our monoclinic material.

The constitutive matrix equation of a monoclinic material has the following form (see [1],Chap.2) :

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{55} & 0 \\ C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix}. \tag{2.1}$$

In Eq.(2.1) we used Voigt's convention by which the tensional indices are replaced by matrix indices in the expressions of the stress and shear components σ_i and ε_i, $i = \overline{1,6}$. The elements $C_{ij}, i, j = \overline{1,6}$ of the stiffness matrix from (2.1) represent the elasticities of the monoclinic material.

The fields equations of a monoclinic material in antiplane strain equilibrium state are:
-displacement Eqs.

$$u_1 = u_2 = 0, \;\; u_3 = u_3(x_1, x_2); \tag{2.2}$$

-strain Eqs.

$$\varepsilon_{11} = \varepsilon_{22} = \varepsilon_{33} = \varepsilon_{12} = 0, \; \varepsilon_{13} = \frac{1}{2}u_{3,1}, \; \varepsilon_{23} = \frac{1}{2}u_{3,2}; \tag{2.3}$$

-stress Eqs.

$$\sigma_{11} = \sigma_{22} = \sigma_{33} = 0, \; \sigma_{23} = C_{44}u_{3,2} + C_{45}u_{3,1}, \; \sigma_{31} = C_{45}u_{3,2} + C_{55}u_{3,1}. \tag{2.4}$$

Consequently, Cauchy's first two equations are identically satisfied and the third equation becomes:

$$\sigma_{13,1} + \sigma_{23,2} = 0. \tag{2.5}$$

Using Eqs.$(2.4)_1$ and (2.5) the equilibrium equation satisfied by u_3 can be written in the following form:

$$u_{3,22} + 2\frac{C_{45}}{C_{44}}u_{3,12} + \frac{C_{55}}{C_{44}}u_{3,11} = 0. \tag{2.6}$$

In the case of a *prestressed* monoclinic material by initially applied stress $\overset{\circ}{\sigma}$ acting in the direction of the reinforcing fibers the incremental fields equations are obtained form Eq.(2.6) replacing the elasticity C_{55} by the coefficient $C_{55}+\overset{\circ}{\sigma}$

Equation (2.6) can be factorized as follows:

$$(\frac{\partial}{\partial x_2}-\mu_1\frac{\partial}{\partial x_1})(\frac{\partial}{\partial x_2}-\mu_2\frac{\partial}{\partial x_1})u_3=0, \tag{2.7}$$

where μ_1 and μ_2 are constant quantities.

The specific strain energy w of our monoclinic composite in antiplane state has the following expression:

$$w=(C_{55}+\overset{\circ}{\sigma})\varepsilon_{13}^2+2C_{45}\varepsilon_{12}\varepsilon_{23}+C_{44}\varepsilon_{23}^2. \tag{2.8}$$

Since the reference configuration of the material is locally stable, w is a quadratic form. Hence the elasticities C_{44}, C_{55} and $C_{55}+\overset{\circ}{\sigma}$ must satisfy the following restrictions:

$$C_{44}>0,\ C_{55}>0,\ C_{44}(C_{55}+\overset{\circ}{\sigma})-C_{45}^2>0. \tag{2.9}$$

The Eqs.(2.7) and (2.8) are equivalent and we have the following relations between the coefficients μ_1 and μ_2 :

$$\mu_1+\mu_2=-2\frac{C_{45}}{C_{44}},\ \mu_1\mu_2=\frac{C_{55}+\overset{\circ}{\sigma}}{C_{44}}. \tag{2.10}$$

Hence μ_1 and μ_2 are the roots of the algebraic equation

$$\mu^2+2\frac{C_{45}}{C_{44}}\mu+\frac{C_{55}+\overset{\circ}{\sigma}}{C_{44}}=0. \tag{2.11}$$

Accordingly, we get

$$\mu_{1,2}=\frac{1}{C_{44}}(-C_{45}\pm i\sqrt{C_{44}(C_{55}+\overset{\circ}{\sigma})-C_{45}^2}). \tag{2.12}$$

From $(2.9)_3$ we obtain that μ_1 and μ_2 are conjugate complex numbers.We denote by μ the root μ_1 and we have

$$\mu=\mu_1=\overline{\mu}_2, \tag{2.13}$$

where bar represents complex conjugation.

The differential Eq.(2.7) satisfied by u_3 becomes

$$(\frac{\partial}{\partial x_2} - \mu\frac{\partial}{\partial x_1})(\frac{\partial}{\partial x_2} - \overline{\mu}\frac{\partial}{\partial x_1})u_3 = 0. \tag{2.14}$$

Introducing the complex variable z_3 defined as below

$$z_3 = x_1 + \mu x_2, \tag{2.15}$$

the above differential equation (2.14) takes the following form:

$$\frac{\partial^2 u_3}{\partial z_3 \partial \overline{z}_3} = 0. \tag{2.16}$$

Thus we conclude that the antiplane elastic state can be represented in the following way due to Guz (see[2]):

$$u_3 = 2\mathrm{Re}f(z_3)$$

$$\sigma_{13} = 2\mathrm{Re}(\rho_1 f'(z_3)) \ \text{ with } \ \rho_1 = C_{55}+ \overset{\circ}{\sigma} +C_{45}\mu$$
$$\sigma_{23} = 2\mathrm{Re}(\rho_2 f'(z_3)) \ \text{ with } \ \rho_2 = C_{45} + C_{44}\mu, \tag{2.17}$$

$f_3 = f_3(z_3)$ being an arbitrary analytic function depending by the complex variable z_3.

Introducing the analytic function

$$\Phi(z_3) = \rho_2 f(z_3) \tag{2.18}$$

we get the following representation of the incremental fields by the complex potentials $\Phi(z_3)$ and $\Psi(z_3)$:

$$u_3 = 2\mathrm{Re}\, \rho_2^{-1}\Phi(z_3)$$

$$\sigma_{13} = 2\mathrm{Re}\, \mathrm{q}\Psi(z_3);\ \sigma_{23} = 2\mathrm{Re}\, \Phi(z_3), \tag{2.19}$$

where

$$\Psi(z_3) = \Phi'(z_3) \text{ and } q = \frac{\rho_1}{\rho_2}. \tag{2.20}$$

3 Antiplane crack in a prestressed monoclinic composite

We assume that the material contains a crack. Let be $k = k(x_1)$ the given value of the applied incremental tearing stress antisymmetrical distributed relative to the plane $x_2 = 0$ and having the direction of x_3 axis. Accordingly to the assumptions made in previous Sections the incremental state of the composites will be an antiplane state relative to x_1x_2 plane.

The involved nominal stress σ_{23} must satisfy the following boundary conditions on the two faces of the crack:

$$\sigma_{23}(x_1, 0^+) = \sigma_{23}(x_1, 0^-) = -k(x_1), \text{ for } |x_1| < a, \tag{3.1}$$

where $k = k(x_1)$ is the given value of the applied incremental tearing tangential surface stress and plus (+) and minus (-) signs refer to the boundary values on the upper and the lower faces of the crack. Also we must have the following far fields conditions:

$$\lim_{r\to\infty} \{u_3(x_1, x_2), \sigma_{13}(x_1, x_2), \sigma_{23}(x_1, x_2)\} = 0,$$
$$r = \sqrt{x_1^2 + x_2^2}. \tag{3.2}$$

From the conditions (3.1) and (3.2) we obtain that on the two faces of the crack, the complex potential $\Psi(z_3)$ define in (2.20), must to satisfy the equations:

$$\Psi^+(x_1) + \overline{\Psi}^-(x_1) = -k(x_1),$$
$$\Psi^+(x_1) + \overline{\Psi}^+(x_1) = -k(x_1), \text{ for } |x_1| < a. \tag{3.3}$$

Using Eqs. (2.19), (2.20) and the conditions (3.2) we obtain:

$$\lim_{|z_3|\to\infty} \{\Phi(z_3), \Psi(z_3)\} = 0. \tag{3.4}$$

Adding and substracting Equations (3.3) we get

$$\left(\Psi^+ + \overline{\Psi}\right)^+ (x_1) + \left(\Psi + \overline{\Psi}\right)^- (x_1) = -2k\,(x_1)\,,$$
$$\left(\Psi - \overline{\Psi}\right)^- (x_1) + \left(\Psi - \overline{\Psi}\right)^- (x_1) = 0, \text{ for } |x_1| < a. \tag{3.5}$$

From the second condition we have

$$\Psi(z_3) = \overline{\Psi}(z_3) \text{ for any } z_3 = x_1 + \mu_1 x_2. \tag{3.6}$$

and taking into account that the first boundary condition(3.5), represents a homogenous Riemann-Hilbert problem (see[3], Chapter 6) the general solution satisfy the far field conditions is

$$\Psi(z_3) = \Phi'(z_3) = -\frac{X(z_3)}{2\pi i}\int_{-a}^{a} \frac{k(t)}{X^+(t)(t - z_3)} dt. \tag{3.7}$$

In above relation $X = X(z)$ represent *Plemelj function* and it has the following expression:

$$X(z) = \frac{1}{\sqrt{z^2 - a^2}} \text{ and } X^+(t) = \frac{1}{i\sqrt{a^2 - t^2}}, \text{ for } t \in (-a, a). \tag{3.8}$$

4 Asymptotic behavior of the incremental fields

For a small neighborhood of the crack tip consider that $x_1 \approx a,\ x_2 \approx 0$ and obviously we have that $z_3 \approx a$. Consequently the Plemelj functions may be approximated by

$$\sqrt{z_3^2 - a^2} = \sqrt{2ar}\chi(\varphi) \tag{4.1}$$

with

$$\chi(\varphi) = \sqrt{\cos\varphi + \mu \sin\varphi} \tag{4.2}$$

and we obtain the following values of the involved complex potentials:

$$\begin{aligned} \Psi(z_3) &= -\frac{K_{III}}{2\sqrt{2\pi r}}\frac{1}{\chi(\varphi)}, \\ \Phi(z_3) &= -K_{III}\sqrt{\frac{r}{2\pi}}\chi(\varphi). \end{aligned} \tag{4.3}$$

In the Equations (4.3) K_{III} represents *the stress intensity factor* corresponding to the third fracture mode and it has the following form:

$$K_{III} = \frac{1}{\pi a}\int_{-a}^{a} k(t)\sqrt{\frac{a+t}{a-t}}dt. \tag{4.4}$$

This quantity has the same expression as in the classical theory of brittle fracture mechanics of elastic materials without initial stresses. Taking into account the representation of incremental fields (2.19)-(2.20) and equations (4.3)-(4.4) we obtain the following asymptotical representation:

$$\begin{aligned} u_3 &= -K_{III}\sqrt{\frac{r}{2\pi}} Re\frac{\chi(\varphi)}{\rho_2} \\ \sigma_{13} &= -\frac{K_{III}}{\sqrt{2\pi r}} Re\frac{q}{\chi(\varphi)} \text{ and } \sigma_{23} = -\frac{K_{III}}{\sqrt{2\pi r}} Re\frac{1}{\chi(\varphi)}. \end{aligned} \tag{4.5}$$

In that follows we shall assume that the given incremental stresses acting on the crack forces have a constant value; *i.e*

$$k(x_1) = k = \text{const.} > 0, \text{ for } \mid x_1 \mid < 0. \tag{4.6}$$

The expressions of the complex potentials and of the incremental fields become

$$\Psi\left(z_3\right)=\frac{\kappa}{2}\left(\frac{z_3}{\sqrt{z_3^2-a^2}}-1\right),\Phi\left(z_3\right)=\frac{\kappa}{2}\left(\sqrt{z_3^2-a^2}-z_3\right),$$
$$u_3=\kappa Re\rho_2^{-1}\left(\sqrt{z_3^2-a^2}-z_3\right),$$
$$\sigma_{13}=\kappa Req\left(\frac{z_3}{\sqrt{z_3^2-a^2}}-1\right),\ \sigma_{23}=\kappa Re\left(\frac{z_3}{\sqrt{z_3^2-a^2}}-1\right). \quad (4.7)$$

Taking into account the Equations (4.1)-(4.2) and $(4.7)_{3-5}$ we obtain the following asymptotic expressions of the incremental fields:

$$u_3=k\sqrt{2ar}Re\frac{\sqrt{\cos\varphi+\mu\sin\varphi}}{\rho_2}$$
$$\sigma_{13}=\frac{k\sqrt{a}}{\sqrt{2r}}Re\frac{q}{\sqrt{\cos\varphi+\mu\sin\varphi}},\quad \sigma_{23}=\frac{k\sqrt{a}}{\sqrt{2r}}Re\frac{1}{\sqrt{\cos\varphi+\mu\sin\varphi}}. \quad (4.8)$$

We observe that the incremental displacement u_3 behaves like $\sqrt{r}$ and the stresses σ_{13} and σ_{23} like $1/\sqrt{r}$ near the crack tip. Now, we shall analyze the behavior of the incremental fields of the crack line $x_2=0$ behind and ahead the crack. From Equations (2.12) and $(2.17)_3$ we obtain that ρ_2 is an imaginary and it has the following expression:

$$\rho_2=i\sqrt{C_{44}(C_{55}+\overset{o}{\sigma})-C_{45}^2}. \quad (4.9)$$

According to Equations $(4.8)_1$ and(4.9) we have the following asymptotic expression of the incremental displacement u_3 for a monoclinic composite:

$$u_3\left(x_1,x_2\right)=\frac{k}{\sqrt{C_{44}(C_{55}+\overset{o}{\sigma})-C_{45}^2}}Im(z_3-\sqrt{z_3^2-a^2}). \quad (4.10)$$

Consequently we get the following asymptomatic expression of the incremental displacement u_3 of the crack line

$$u_3(x_1,0)=0,\quad \text{for}|x_1|>a,$$
$$u_3\left(x_1,0^+\right)=\frac{\kappa}{\sqrt{C_{44}(C_{55}+\overset{o}{\sigma})-C_{45}^2}}\sqrt{a^2-x_1^2},\ \text{for}\ |x_1|<a,$$
$$u_3\left(x_1,0^-\right)=-\frac{\kappa}{\sqrt{C_{44}(C_{55}+\overset{o}{\sigma})-C_{45}^2}}\sqrt{a-x_1^2},\ \text{for}\ |x_1|>a. \quad (4.11)$$

In the particular case of an orthotropic material, with the elasticity $C_{45} = 0$, the same expressions of u_3 rest valid.

Concerning the above results we make the following remarks:

1. The antiplane displacements u_3 of the crack lines behind and ahead the crack is zero.
2. The antiplane displacements of the two faces of the crack are symmetric relative to the line $x_2 = 0$.

The same conclusions were obtain also for an orthotropic material (see[5]). We explain these results due to the fact that the tearing loads are antisymmetricaly applied relative to the plane x_1, x_2 representing the symmetry plane for our monoclinic material.

5 Griffith's energy criterion. Resonance

In this Section we shall use an energetic criterion due to Griffith, necessary to find the critical value of incremental tearing stress which produces crack propagation. Also, following Soós (see[6]) we shall study the resonance phenomenon.

Let us denote by $U(a)$ the elastic energy of the body when the length of the crack is $2a$ and let $U(a + \delta a)$ its elastic energy when the length of the crack is $2(a + \delta a)$. According to Griffith's criterium a necessary condition for crack propagation is that the change in strain energy must satisfy the inequality

$$U(a) - U(a + \delta_a) \geq 4\gamma\delta a, \tag{5.1}$$

γ representing the specific surface, energy of the body. Expanding in Taylor's series the function $U(a + \delta_a)$ and neglecting the terms of higher order then δ_a, we get

$$\delta U = U(a) - U(a + \delta a) = -\frac{\partial U}{\partial a}(a)\delta a. \tag{5.2}$$

The strain energy release rate $G(a)$ is defined by the equation

$$G_{III}(a) = -\frac{1}{2}\frac{\partial U}{\partial a}(a). \tag{5.3}$$

Consequently, from Eqs. (5.2.) and (5.3) the crack instability condition (5.1) takes the equivalent form called Griffith's energy criterion

$$G_{III}(a) \geq 2\gamma. \tag{5.4}$$

The energy release rate $G_{III}(a)$, which may be regarded as the force tending to open the crack, represents the work done in this process by the incremental nominal stress $\sigma_{23}(\delta a - t, 0)$ acting by incremental displacement $u_3(t, 0^+)$ provided that δa is very small such that in the limit as $\delta_1 \longrightarrow 0$ the conditions $u_2(t, 0^+) \longrightarrow u_2(\delta - t', 0^+)$ and $t \longrightarrow t'$ are fulfilled. The involved strain energy release rate $G_{III}(a)$ can be obtained using Irwin's relation (see for instance [7],[8]):

$$G_{III}(a)\delta a = \int_0^{\delta a} \sigma_{23}(\delta a - t, 0) u_3(t, 0^+) dt. \tag{5.5}$$

Using the Eqs. (4.8) and (4.9) we obtain the following asymptotic behavior of the incremental displacement and stress fields u_3 and σ_{23} :

$$u_3 = \frac{k}{\sqrt{C_{44}(C_{55} + \overset{o}{\sigma} - C_{45}^2}} \sqrt{2ar} Im \sqrt{\cos\varphi + \mu \sin\varphi}$$

$$\sigma_{23} = \frac{k\sqrt{a}}{\sqrt{2r}} Re \frac{1}{\sqrt{\cos\varphi + \mu \sin\varphi}}. \tag{5.6}$$

The incremental stress $\sigma_{23}(\delta a - t, 0)$ can be obtained taking in its asymptotic expression $(4.7)_5$

$$r = \delta a - t > 0 \text{ and } \varphi = 0.$$

We get :

$$\sigma_{23}(\delta a - t, 0) = \frac{k\sqrt{a}}{\sqrt{2(\delta a - t)}}. \tag{5.7}$$

The antiplane incremental displacement $u_3(t, 0^+)$ can be obtained taking in its asymptotic expression $(4.7)_3$ $r = t > 0$ and $\varphi = \pi$.
We get :

$$u_3(t, 0^+) = \frac{k}{\sqrt{C_{44}(C_{57} + \overset{o}{\sigma}) - C_{45}^2}} \sqrt{2at}. \tag{5.8}$$

Using the Eqs. (5.5),(5.7) and (5.8) we obtain that the strain energy release rate has the following value:

$$G_{III}(a) = \frac{\pi a k^2}{2\sqrt{C_{44}(C_{55} + \overset{o}{\sigma}) - C_{45}^2}}. \tag{5.9}$$

According to Griffith's energy criterion, the crack will propagate if:

$$G_{III}(a) = 2\gamma. \tag{5.10}$$

Hence, the crack propagation starts when the applied tearing stress k reaches its critical value k_c given by the following relation:

$$k_c^2 = k_c^2(\overset{o}{\sigma}) = \frac{4\gamma}{\pi a}\sqrt{C_{44}(C_{55}+\overset{o}{\sigma}) - C_{45}^2}. \tag{5.11}$$

We denote by $\widehat{k}_c$ the critical value corresponding to the case when $\overset{o}{\sigma}= 0$ From Eq (5.11) we obtain :

$$\widehat{k}_c^2 = k_c^2(0) = \frac{4\gamma}{\pi a}\sqrt{\Gamma}, \tag{5.12}$$

where we denoted by

$$\Gamma = C_{44}C_{55} - C_{45}^2.$$

The inequalities (2.9) rest true also in this case for $\overset{o}{\sigma}= 0$ and we have that:

$$C_{44} > 0, C_{55} > 0 \text{ and } C_{44}C_{55} - C_{45}^2 > 0. \tag{5.13}$$

Using the Eqs. (5.11) and (5.12) we obtain :

$$k_c^2(\overset{o}{\sigma}) = \widehat{k}_c^2\sqrt{1 + \frac{C_{44}\,\overset{o}{\sigma}}{C_{44}C_{55} - C_{45}^2}} \tag{5.14}$$

So, we obtain the following conclusion:

$$\begin{aligned} k_c(\overset{o}{\sigma}) > \widehat{k}_c \text{ if } \overset{o}{\sigma}> 0; \\ k_c(\overset{o}{\sigma}) < \widehat{k}_c \text{ if } \overset{o}{\sigma}< 0. \end{aligned} \tag{5.15}$$

Consequently, an initial applied extensional stress $\overset{o}{\sigma}$ acting in the direction of the reinforcing fibers of an monoclinic composite material improves the crack instability and applied compresive stress $\overset{o}{\sigma}< 0$ acting in the fiber direction diminishes the stability of the crack.

Now, can arise the following question :
May exist a critical value $\overset{o}{\sigma}_c$ of the initial applied stress such that when $\overset{o}{\sigma}$ starts from zero to $\overset{o}{\sigma}_c$, the coefficient Γ converges to infinity?
But, from Eq. (5.11) we have that :

$$k_c(\overset{o}{\sigma}) \longrightarrow 0 \text{ if } \overset{o}{\sigma}\longrightarrow \overset{o}{\sigma}^c = -\frac{C_{45}^2 - C_{44}C_{55}}{C_{44}}. \tag{5.16}$$

Hence, when the applied compressive stress $\overset{o}{\sigma}$ reaches the *critical value* .

$$\overset{o}{\sigma}^c = -\frac{C_{45}^2 - C_{44}C_{55}}{C_{44}}, \tag{5.17}$$

the crack becomes completely unstable and it's propagation can start *without* any incremental tearing stress applied on the two faces of the crack.

In what follows we particularize our study regarding the resonance of the pressed fiber reinforced elastic composite to the case of an prestressed elastic orthotropic material; *i.e.*

$$C_{45} = 0. \tag{5.18}$$

Using the above condition (5.18) in Eq. (5.14) we observe that the conclusions (5.15) rest valid. From Eq. (5.11) we obtain that the initial applied compressive stress $\overset{o}{\sigma}$ reaches the critical value.

$$\overset{o}{\sigma}{}^{c} = -C_{55}.$$

We recall that for a fiber reinforced elastic material we have

$$C_{55} = G_{13} \text{ and } G_{13} << E_1.$$

there G_{13} and E_1 represent the shear modulus in x_1x_3 plane and respectively Young's modulus in the direction of x_1 axis.

Hence

$$\overset{o}{\sigma}_c = -G_{13} \text{ and } \mid \overset{o}{\sigma}_c \mid << E_1, \tag{5.19}$$

and we observe that the critical value of compression stress produces only infinitesimal strains in the prestressed composite. The value of the coefficient Γ becomes very large in the domain of infinitesimal strains. If condition (5.21) is fulfilled the complete instability of the crack occurs in our orthotropic material.

Also, using Eq. (5.18) in Eqs. $(2.10)_2$, (2.12) and (5.19) we observe that $\mu = 0$ if $\overset{o}{\sigma}_c = \overset{o}{\sigma}$. Hence, if the compressive stress $\overset{o}{\sigma}$ reaches its critical value the differential equation obtained using (5.18) in (2.6) losses its ellipticity. In other words internal instability of the prestressed fiber reinforce orthotropic composite occurs. The *simultaneous* appearance of the internal instability and of the complete instability of the crack are the direct consequences of the internal structure of the fiber reinforced orthotropic composite.

6 Final remarks

The behavior of the incremental fields for a monoclinic composite containing a crack supposed to antiplane tearing stress was studied.
The following main conclusion arises:

- the antiplane incremental displacement of the crack lines behind and ahead the crack are zero, as for an orthotropic composite;

- the antiplane incremental displacement of the two faces are symmetric relative to the line $x_2 = 0$, as for an orthotropic composite.

The value of incremental antiplane stress which produces crack propagation were obtained and we can conclude that an initial applied compressive, respectively extensional, stress acting in the direction of the reinforcing fibers diminishes, respectively improves, the crack instability. In our opinion the results are plausible and in good agreement with the experimental crack behavior.

We conclude that just due to its internal structure in a fiber reinforced composite material can occur dangerous situation if the initial applied forces are not adequately limited. To avoid such dangerous situation of resonance phenomenon leading to the total rupture of composite, the initial applied compression force must drastically limited.

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"Ovidius" University of Constanta,
Faculty of Mathematics and Informatics,
8700 Constantza,
Institute of Statistics and Applied Mathematics of the Romanian Academy,
Romania
e-mail: mcraciun@univ-ovidius.ro