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DETERMINATION OF RELIABILITY BOUNDS FOR STRUCTURAL SYSTEMS USING LINEAR PROGRAMMING

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Abstract

In reliability analysis of structural systems, what is of interest is intervals containing the survival probability when exact values cannot be obtained. There are analitical formulas to determinate the bounds of the system survival probability for series systems by employing bi- and higher-order component probabilities. For parallel systems, the bounds of the system survival probability cannot be found analytically only from marginal probabilities (first order component probabilities). Linear programming can be used to calculate the bounds of survival (or failure) system probability for any system with any amount of information available on the component probabilities. This paper considers two-state (intact or failure) component systems and uses the probabilities associated to these events as decision variables. The linear objective function is a sum containing some of these probabilities, with linear restrictions given by the individual probabilities and/or joint component probabilities and by the axioms of probability. The lower bound of the system probability is obtained as the minimum of the objective function and the upper bound is obtained as its maximum.

1 Introduction

The reliability of a structural system is its ability to fulfill its design purpose for some specified reference period (Toft-Christensen and Murotsu, 1986).

The basic variables that emerge during the design process (loads, material properties, geometrical characteristics) are modelled as random variables.

Failure occurs when the load effects (L) exceeds the resistance (R) of the structure and can be derived by considering the probability density functions of L and R. The main goal for the safety of the structure is to guarantee an $R > L$ scenario throughout the design life of the structure.

The probability of failure of structural systems composed of structural elements is calculed by forming failure functions (limit state functions) for the components. The component failure functions are then combined to form a multidimensional limit surface for the system and its failure criteria. The probability of failure is subsequently calculated through multidimensional volume integration over the relevant space.

If $g(X)$ $=g(X_1, ..., X_n)$ denote the *failure function,* the probability of component failure is defined by

$$P_F = P(g(X) \leq 0) = \int_{g(X)\leq 0} f_X(x)dx,$$

and the reliability is

$$P_S = 1 - P_F = P(g(X) > 0) = \int_{g(X)>0} f_X(x)dx,$$

where $f_X(x)$ is the probability density function (pdf) of random vector X of basic variables.

For example, assuming that R and L are normal, statistically independent random variables, reliability can be defined as the survival probability of component

$$P_S = P\left(R > L\right) = \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{L<R} f_R\left(r\right) f_L\left(l\right) drdl$$

where $f_R(r)$ is the pdf of resistance variable and $f_L(l)$ is the pdf of the load effect variable. The *safety index* can now be defined as

$$\beta = \Phi^{-1}\left(P_S\right) = \Phi^{-1}\left(1 - P_F\right),$$

with Φ - the cumulative normal distribution function (cdf).

Reliability is synonimous with operational safety of the system. It underlines some performance indicators that describe the well-functioning of the system.

In what follows structural systems are assumed to be made of two-state components (component is intact or component failed).

Given a system with n components, a *state variable* can be assigned to each component i (Vãduva, 2003)

$$Z_i = \begin{cases} 1, & \text{if component } i \text{ is operational} \\ 0, & \text{otherwise .} \end{cases}$$

Similarly, a state variable called *structural function of the system* can be assigned to the system:

$$\varphi(Z) = \varphi(Z_1, Z_2..., Z_n) = \begin{cases} 1 & \text{if the system is operational} \\ 0 & \text{otherwise.} \end{cases}$$

The structural function can be determined when the *topology of the system,* i.e. the arrangement of the components, is known (components in series, in parallel or a combination of these).

The system S is of the type " k of n " if it fails when at least k its components fail. In this case, there are $m = C_n^k$ subsets, C_j, of k of the n components of S, and the structural function is defined as

$$\varphi(Z) = \min_{1\leq j\leq m} \max_{i\in C_j} \{Z_i\}$$

When k=1, the components of the system are in parallel (the system is called *parallel system)* and

$$\varphi(Z) = \min_{1\leq j\leq n} \{Z_j\} = 1 - (1 - Z_1)(1 - Z_2)...(1 - Z_n).$$

Parallel systems are *the redundant systems*. In engineering terminology, *redundancy* is defined as the capacity of structural systems to continue to carry loads following the failure of one or more of their components

When k=n, the components of the system are in series (the system is called *series system)* and

$$\varphi(Z) = \max_{1\leq j\leq n} \{Z_j\} = Z_1 Z_2 ... Z_n.$$

When the components of the system are statistically independent, the reliability of the system is (Lixandroiu, 2001)

$$P_S = E\left[\varphi(X)\right],$$

where $E[.]$ is mean value.

The reliability issue can be approached as follows: given the survival probabilities of the components, denoted by P_{S_1},...,P_{S_n}, find the function $h(Z_1, Z_2, \ldots\ldots, Z_n)$ such that the reliability of the system S is

$$P_S = h\left(P_{S_1}, ...P_{S_n}\right).$$

2 Bounds for the survival and failure probabilities

Consider the case when the performance function g depends liniarly on R and L, ($g = R - L$), and R and L are assumed to be random variabiles. Let $E_i = \{R_i < L_i\}$ be the event of failure of the i-th component. The exact survival and failure probabilities depend on the correlation between the failure types and are difficult to caculate in general.

In the failure tree, let the path j denote the path that leads to the failure of the system. The probability of failure of this path is the product of conditional probabilities on the path (Ayyub and Ibrahim, 1989):

$$P\,(path\ j) = \prod_{i=1}^{k} P(E_{ji})$$

where $P(E_{ji})$ is the probability of event i occuring on path j,conditional on events $E_{j1}, E_{j2}, ..., E_{ji-1}$ occuring, meaning that path j occured if the intersection of the corresponding events occured:

$$path\ j = \bigcap_{i=1}^{k} P(E_{ji}).$$

The lower bound of the probability failure of a redundant system is the probability of the most likely path to lead to failure, that is, the highest probability of occurance of a failure path. The upper bound is the probability of the union of all failure paths of all components defining the intact structure:

$$\max_{1 \le j \le r} P(path\ j) \le P_F \le 1 - \prod_{j=1}^{r} \left[1 - P\,(path\ j)\right]$$

where r is the number of failure paths in the failure tree. The upper bound corresponds to the case of uncorreled failure paths, while the lower bound corresponds to perfectly correlated failure paths.

The estimation method of the probability of failure of a system based on the probability of failure of all single failure modes is called *one-order bounds,* or *Boole's bounds,* estimation. These bounds are, for a series system (Song and Der Kiureghian, 2003)

$$\max_{1 \le i \le n} P_i \le P_F = P\left(\bigcup_{i=1}^{n} E_i\right) \le \min\left\{\sum_{i=1}^{n} P_i, 1\right\}$$

and for a parallel system

$$\max\left\{0, \sum_{i=1}^{n} P_i - (n-1)\right\} \le P_F = P\left(\bigcap_{i=1}^{n} E_i\right) \le \min_{1 \le i \le n}\left(P_i\right).$$

This is a rather easy procedure, but it yields a relatively large interval.

Ditlevsen derived a smaller interval for the probability of failure of a structural system using all single-mode probabilities of failure and all two-mode joint failure probabilities, the so-called *two-order bounds (Ditlevsen's bounds)*, (Ditlevsen, 1979). For a series system, these bounds are

$$P_1 + \sum_{i=2}^{n} \max \left\{ P_i - \sum_{j=1}^{i-1} P_{ij}; 0 \right\} \leq P_F = P\left(\bigcup_{i=1}^{n} E_i \right) \leq \sum_{i=1}^{n} P_i - \sum_{i=2}^{n} \max_{j<i} \; P_{ij}. \tag{1}$$

The bounds of the interval are significanly improved if first, second, and higher order probabilities are taken into acount (Zhang, 1993). Also, the midpoint of the interval can be used as an approximation for the failure probability (Feng and Yang, 1991).

The *three-order bounds* are:

$$P\left(E_1 \cup E_2\right) + \sum_{i=3}^{n} \max \left\{ P_i - \sum_{j=1}^{i-1} P_{ij} + \max_{1 \leq l \leq i-1} \sum_{j \neq l, j=1}^{i-1} P_{ijl}; 0 \right\} \leq P_F \leq$$

$$\leq P\left(E_1 \cup E_2\right) + \sum_{i=3}^{n} \left[P_i - \max_{j<l,\; 1 \leq l \leq i-1} \left\{ P_{il} + P_{ij} - P_{ilj} \right\} \right] \tag{2}$$

where: $P_{ij}{=}P(E_iE_j)$, $P_{ilj}{=}P(E_iE_lE_j)$.

The *k-order bounds* $(0 < k < n)$ can be obtained analogously to the three-order bounds:

$$P\left(E_1 \cup ... \cup E_{k-1}\right) + \sum_{i=k}^{n} \max \left\{ \widetilde{P}_i; 0 \right\} \leq P_F$$

$$P_F \leq P\left(E_1 \cup ... \cup E_{k-1}\right) + \sum_{i=k}^{n} \min_{I_{k-1}} \left(P\left(E_i \bigcap_{s=1}^{k-1} \overline{E_{r_s}} \right) \right), \tag{3}$$

where: I_{k-2} is a set of $k-2$ integers between 1 and $i-1$, and

$$\widetilde{P}_i = P\left(E_i \bigcap_{s=1}^{k-2} \overline{E_{l_s}} \right) - \max_{I_{k-2}} \sum_{s=1}^{i-k+1} P\left[E_i \left(\bigcap_{s=1}^{k-2} \overline{E_{l_s}} \right) E_{j_s} \right].$$

Since the series system state is characterized by union operations, $E_{sys} = \bigcup_{i=1}^{n} E_i$, and the parallel system state by intersection operations, $E_{sys} = \bigcap_{i=1}^{n} E_i$, a parallel system can be transformed into a series system using De Morgan's rule: $\overline{\bigcap_{i=1}^{n} E_i} = \bigcup_{i=1}^{n} \overline{E_i}$. In this way the bounds for a parallel systems can be calculated using equations (1), (2) and (3).

3 Derivation of two- and three-order failure probabilities for series systems

For a series system, if the safety margin corresponding to the i-th failure mode is denoted by M_i, (Zang, 1991) then

$$M_i = \begin{cases} \leq 0 & \text{if the } i\text{-th failure mode occurs} \\ > 0 & \text{otherwise} \end{cases} .$$

Let E_i be the i-th failure mode occurence event, $i = 1, 2, ..., m$, and let E be the event characterizing the system state.

Then the failure probabilities P_i, P_{ij}, P_{ijk} are given by the following equations:

$$P_i = P\left(M_i < 0\right) = P(E_i) = \int_{-\infty}^{\infty} f\left(m_i\right) dm_i,$$

$$P_{ij} = P\left(\left(M_i < 0\right) \cap \left(M_j < 0\right)\right) = P(E_i E_j) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{ij}\left(m_i, m_j\right) dm_i dm_j,$$

$$P_{ijk} = P\left(\left(M_i < 0\right) \cap \left(M_j < 0\right) \cap \left(M_k < 0\right)\right) = P(E_i E_j E_k) =$$
$$\int_{-\infty}^{0\infty} \int_{-\infty}^{0} \int_{-\infty}^{0} f_{ijk}\left(m_i, m_j, m_k\right) dm_i dm_j dm_k,$$

where: f_i, f_{ij}, f_{ijk} are the pfd's of M_i, (M_i, M_j) and (M_i, M_j, M_k), respectively. When M_i, M_j and M_k are normally distributed, one can derive analitical expressions for the integrals above, but the calculations are very difficult. Song (1992) presents a method for numerical integration to evaluate these integrals.

4 Evaluation of the bounds of the system reliability using linear programming

For a series system having n two-state components, the sample space of the component events has 2^n mutually exclusive and collectively exhaustive (MECE) events. Table 1 gives the intersections of the events (paths), the state vectors, the MECE events, and the corresponding elementary probabilities when n=3 (the case of a series system associated with the parallel system in Example 2 in the next section).

Table 1

Paths	$E_1E_2E_3$	$E_1E_2\overline{E_3}$	$E_1\overline{E_2}E_3$	$\overline{E_1}E_2E_3$
State	000	001	010	100
MECE	e_1	e_2	e_3	e_4
Probabilities	p_1	p_2	p_3	p_4
Paths	$E_1\overline{E_2E_3}$	$\overline{E_1}E_2\overline{E_3}$	$\overline{E_1E_2}E_3$	$\overline{E_1E_2E_3}$
State	011	101	110	111
MECE	e_5	e_6	e_7	e_8
Probabilities	p_5	p_6	p_7	p_8

The probabilities vector $p = (p_1, ..., p_{2^n})$ is the vector of decision variabiles in the linear programming (LP) problem. The probabilities p must satisfy the linear constraints

$$\sum_{i=1}^{2^n} p_i = 1,\; 0 \leq p_i \leq 1, (\forall)\, i = \overline{1, 2^n},$$

according to the basic axioms of probability, and the constraints concerning the information on the uni-, bi- and three-component probabilities, if they exist. When n=3, these constraints are given by the following equations:

$$\begin{gathered}
p_1 + p_2 + p_3 + p_5 = P(E_1) = P_1, \\
p_1 + p_2 + p_4 + p_6 = P(E_2) = P_2 \\
p_1 + p_3 + p_4 + p_7 = P(E_3) = P_3 \\
p_1 + p_2 = P(E_1E_2) = P2_{12}, \\
p_1 + p_3 = P(E_1E_3) = P2_{13}, \\
p_2 + p_3 = P(E_2E_3) = P2_{23}.
\end{gathered}$$

The survival probability $h(p) = p_{2^n}$ or the failure probability of the structural system $1 - h(p) = 1 - p_{2^n}$ may both be considered as objective function. Thus, one obtains the LP problem *opt* $h(p)$ subject to the constraints (Song and Der Kiureghian, 2003):

$$\begin{cases}
\sum\limits_{r: e_r \subseteq E_i} p_r = P(E_i) = P_i & i = \overline{1, n} \\
\sum\limits_{r: e_r \subseteq E_i E_j} p_r = P(E_iE_j) = P2_{ij} & 1 \leq i < j \leq n \\
\sum\limits_{r: e_r \subseteq E_i E_j E_k} p_r = P(E_iE_jE_k) = P3_{ijk} & 1 \leq i < j < k \leq n
\end{cases}$$

The lower and upper bound of survival probability given by the function h are obtained when *opt* is *min* or *max*, respectively.

5 Numerical results

The following examples consider series and parallel systems and calculate the bounds for the reliability and the failure probability of these systems. The numerical results are in perfect concordance with the numerical results obtained by other authors.

Example 1. For a series system with 4 components (Zhang, 1993) having the safety index vector $\beta = (0.6, 0.8, 1, 1.2)$, the marginal and joint bi- and three-component probabilities are calculated using the equations:

$$P(E_i) = \int_{-\infty}^{-\beta_i} f(t)dt = \Phi^{-1}(\beta_i), (\forall)\, i = \overline{1,4},$$

$$P_{ij} = P(E_iE_j) = \int_{-\infty}^{-\beta_i} \Phi\left(\frac{\beta_i - \rho_{ij} t}{\sqrt{1-\rho_{ij}^2}}\right) f(t)dt, 1 \le i < j \le 4$$

$$P_{ijk} = P(E_iE_jE_k) = \int_{-\infty}^{\infty} f(t) \prod_{s=\{i,j,k\}} \Phi\left(\frac{\beta_s - \lambda_s t}{\sqrt{1-\lambda_s^2}}\right) dt,$$

where $f(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$, Φ is the standard normal cdf and $\lambda_1, \lambda_2, \lambda_3$ and λ_4,are given by the Dunnett-Sobel decomposition (Dunnett and Sobel, 1955) of the correlation matrix ρ if this decomposition is possible, i.e. $\rho_{ij} = \lambda_i\lambda_j, |\lambda_i| \le 1$, for all $i = \overline{1,4}, \quad i \ne j$. The results are:

$$P = \begin{pmatrix} 0.2742531178 \\ 0.2118553986 \\ 0.1586552539 \\ 0.1150696702 \end{pmatrix}, \quad \text{and for } \rho = \begin{pmatrix} 1 & 0.855 & 0.8075 & 0.76 \\ 0.855 & 1 & 0.765 & 0.72 \\ 0.8075 & 0.765 & 1 & 0.68 \\ 0.76 & 0.72 & 0.68 & 1 \end{pmatrix},$$

$$\text{it follows that } \lambda = \begin{pmatrix} 0.9500006515 \\ 0.9000000041 \\ 0.8500000039 \\ 0.7999999652 \end{pmatrix} \text{ and}$$

$P2_{12}$=0.1710696401 , $P2_{13}$=0.1302165521 , $P2_{14}$=0.0952591086 ,
$P2_{23}$=0.1092029619 , $P2_{24}$=0.0812099041 , $P2_{34}$=0.0656607765 ,
P123=0.1018319141, P124=0.076338052, P134=0.0624301361,
P234=0.0563939207 .

The corresponding bounds are listed in Table 2.

Table 2

Probability of	Bounds			
	Boole		Ditlevsen	
	lower	upper	lower	upper
survival	0.2401665595	0.7257468822	0.5860999381	0.6849611238
failure	0.2742531178	0.7598334405	0.3150388762	0.4139000619

Probability of	Bounds			
	Zang		LP	
	lower	upper	lower	upper
survival	0.6489547602	0.6521854007	0.6489547602	0.6521854007
failure	0.3478145993	0.3510452398	0.3478145993	0.3510452398

Example 2. Let us consider now a parallel system with 3 components (Ang and Tang, 1984). For this system, the load (L) and the resistance (R) are normally distributed, $L \hookrightarrow N(\mu_L, \sigma_L)$ and $R \hookrightarrow N(\mu_R, \sigma_R)$, with a correlation coefficient $\rho = 0.44$ and coefficients of variation

$$V_L = \frac{\sigma_L}{\mu_L} = 0.25, V_R = \frac{\sigma_R}{\mu_R} = 0.15.$$

The following probabilities can be calculated:

$$P = (P_i), \quad P_i = 0.01003, \quad i = \overline{1,3}, \quad P2 = (P2_{ij}), P2_{ij} = 0.00102113, \\ 1 \leq i < j \leq 3.$$

The series system can be obtained by using De Morgan's rule, allowing the calculation of the bounds for the survival and failure probabilities. The results obtained after returning to the initial parallel system are shown in Table 3 .

Table 3

Probability of	Bounds			
	Boole		Ditlevsen&LP	
	lower	upper	lower	upper
survival	0.98997	0.96991	0.97196	0.97298
failure	0.01003	0.03009	0.02702	0.02804

Example 3. Consider a Daniels' parallel system with 4 wires (adapted from Song and Der Kiureghian, 2003) under following conditions: a) the wires are perfectly britle and have identical, deterministic elastic moduli; b) the wire strengths are statistically independent and identically Weibull-distributed random variables with cdf $F(x) = 1 - e^{-\lambda x^\gamma}, \quad x \geq 0$, and c) the load L is deterministic and equally distributed among the surviving wires. Let X_1,X_2,.X_3 and .X_4 denote the random strengths of the 4 wires and let $X_{(1)} \leq X_{(2)} \leq X_{(3)} \leq X_{(4)}$, denote the set of wire strengths in increasing order.

If $E_i = \left\{X_{(i)} \leq \frac{L}{4-i+1}\right\}, i = \overline{1,4}$, and $b_i = F\left(\frac{L}{i}\right)$, then the exact failure probability of system is

$$P_F = n! \det\left(P\left(\bigcap_{i=1}^{4} E_i\right)\right), \text{ where } P\left(\bigcap_{i=1}^{4} E_i\right) = \begin{pmatrix} b_4 & \frac{b_4^2}{2!} & \frac{b_4^3}{3!} & \frac{b_4^4}{4!} \\ 1 & b_3 & \frac{b_3^2}{2!} & \frac{b_3^3}{2!} \\ 0 & 1 & b_2 & \frac{b_2^2}{2!} \\ 0 & 0 & 1 & b_1 \end{pmatrix}.$$

For $\gamma = 20, \lambda = 0.01, L = 4$, the failure probability is P_F =0.0392074604. The Boole, Ditlevsen and LP (for the associated series system) bounds for the survival and failure probabilities can be calculated using the same values for the parameters. The uni-component probabilities for the series system calculated by the equations

$$P_i = 1 - F\left(\frac{L}{5-i}\right) = 1 - \sum_{r=1}^{4} C_4^r F^r\left(\frac{L}{5-i}\right)\left[1 - F\left(\frac{L}{5-i}\right)\right]^{4-r},\ i = \overline{1,4},$$

are P_1=0.0392105608 , P_2=0.99969838872 , P_3=1 and P_4=1, and the bi-component probabilities calculated from the equations

$$P_2 = P\left(E_i E_j\right) = \sum_{l=5-i}^{4} \sum_{s=\max\{0,5-i-l\}} \frac{4!}{l!s!(4-l-s)!}\left[1 - F\left(\frac{L}{5-i}\right)\right]^l \times$$

$$\times\left[F\left(\frac{L}{5-j}\right) - F\left(\frac{L}{5-i}\right)\right] F^{4-l-s}\left(\frac{L}{5-i}\right)$$

are all zero with the exception of $P2_{12}$=0.00000332688. The bounds are shown in Table 4.

Table 4

Probability of	Bounds			
	Boole		Ditlevsen&LP	
	lower	upper	lower	upper
survival	0.9607894392	0.9610910519	0.9610877251	0.9610877251
failure	0.0389089481	0.0392105608	0.0389122749	0.0389122749

Note that the information available gives values for the failure probability very close to the exact value from Daniels' equation.

6 Conclusions

If the marginal and all k-joint component probabilities, $1 \leq k \leq n$, are known, then the constrained system is exactly determined (since the determinant of the constraints matrix is $(-1)^{n-1}$) and the value of the objective function gives the exact survival (failure) probability of the system. The use of LP yields bounds for the survival (failure) probability even when not all bi- and three-component probabilities are known, the accuracy of these bounds depending on the information available.

The major disadvantage of using LP to determine the reliability bounds of structural systems lies in that the size of LP problem grows exponentially with the number of component states (2^n decision variabiles). However, this disadvantage is not critical due to the highly advanced LP algorithms and codes (Song and Der Kiureghian, 2003), at least for systems with at most 17 components.

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