Acta Universitatis Apulensis No. 23/2010
ISSN: 1582-5329 pp. 123-132

SOME APPLICATIONS OF FRACTIONAL CALCULUS OPERATORS TO CERTAIN SUBCLASS OF ANALYTIC FUNCTIONS WITH NEGATIVE COEFFICIENTS

B.A. Frasin

Abstract. The object of the present paper is to derive various distortion theorems for fractional calculus and fractional integral operators of functions in the class $\mathcal{B}_{\mathcal{T}}(j,\lambda,\alpha)$ consisting of analytic and univalent functions with negative coefficients. Furthermore, some of integral operators of functions in the class $\mathcal{B}_{\mathcal{T}}(j,\lambda,\alpha)$ is shown.

2000 *Mathematics Subject Classification*: 30C45.

1.Introduction and definitions

Let $\mathcal{A}(j)$ denote the family of functions of the form:

$$f(z) = z + \sum_{n=j+1}^{\infty} a_n z^n \qquad (j \in \mathbb{N} = \{1,2,3,\cdots\}), \tag{1}$$

which are analytic in the open unit disk $\mathcal{U} = \{z : |z| < 1\}$. A function $f(z)$ belonging to $\mathcal{A}(j)$ is in the class $\mathcal{B}(j,\lambda,\alpha)$ if and only if

$$\text{Re}\left\{\frac{zf'(z) + (2\lambda^2 - \lambda)z^2 f''(z)}{4(\lambda - \lambda^2)z + (2\lambda^2 - \lambda)zf'(z) + (2\lambda^2 - 3\lambda + 1)f(z)}\right\} > \alpha \tag{2}$$

for some $\alpha(0 \le \alpha < 1)$ and $\lambda(0 \le \lambda < 1)$, and for all $z \in \mathcal{U}$.

Let $\mathcal{T}(j)$ denote the subclass of $\mathcal{A}(j)$ consisting of functions of the form:

$$f(z) = z - \sum_{n=j+1}^{\infty} a_n z^n \qquad (a_n \ge 0,\ j \in \mathbb{N}), \tag{3}$$

Further, we define the class $\mathcal{B}_{\mathcal{T}}(j,\lambda,\alpha)$ by

$$\mathcal{B}_{\mathcal{T}}(j,\lambda,\alpha) = \mathcal{B}(j,\lambda,\alpha) \cap \mathcal{T}(j). \tag{4}$$

The class $\mathcal{B}_{\mathcal{T}}(j,\lambda,\alpha)$ was introduced and studied by the author in [3]. The class $\mathcal{B}_{\mathcal{T}}(j,\lambda,\alpha)$ is of special interest because it reduces to various classes of well-known functions as well as many new ones. For example The classes $\mathcal{B}_{\mathcal{T}}(1,0,\alpha)=\mathcal{T}^*(\alpha)$ and $\mathcal{B}_{\mathcal{T}}(1,1,\alpha)=\mathcal{C}(\alpha)$ were first studied by Silverman [10]. The classes $\mathcal{B}_{\mathcal{T}}(j,0,\alpha)=\mathcal{T}^*_{\alpha}(j)$ and $\mathcal{B}_{\mathcal{T}}(j,1,\alpha)=\mathcal{C}_{\alpha}(j)$ were studied Srivastava et al. [13]. The class $\mathcal{B}_{\mathcal{T}}(1,1/2,\alpha)=\mathcal{B}_{\mathcal{T}}(\alpha)$ was studied by Gupta and Jain [4].

In order to show our results, we shall need the following lemma.

Lemma 1. ([3]) *Let the function $f(z)$ be defined by (3). Then $f(z)\in\mathcal{B}_{\mathcal{T}}(j,\lambda,\alpha)$ if and only if*

$$\sum_{n=j+1}^{\infty}\sigma(n,\alpha,\lambda)a_n\le 1-\alpha, \tag{5}$$

where

$$\sigma(n,\alpha,\lambda):=(2\lambda^2-\lambda)n^2+[1+(1+\alpha)(\lambda-2\lambda^2)]n+(1+2\lambda^2-3\lambda)\alpha \tag{6}$$

and $0\le\alpha<1$, $0\le\lambda<1$. *The result is sharp.*

2.Fractional calculus

Many essentially equivalent definitions of fractional calculus (that is fractional derivatives and fractional integrals) have been given in the literature (cf., e.g., [1], [2, Chap. 13], [5],[7], [8], [9], [11, p.28 et. seq.]. We find it to be convenient to recall here the following definitions which are used earlier by Owa [6] (and, subsequently, by Srivastava and Owa [12]).

Definition 1. *The fractional integral of order μ is defined, for a function $f(z)$, by*

$$D_z^{-\mu}f(z)=\frac{1}{\Gamma(\mu)}\int_0^z\frac{f(\zeta)}{(z-\zeta)^{1-\mu}}d\zeta, \tag{7}$$

where $\mu>0$, $f(z)$ is an analytic function in a simply-connected region of the z-plane containing the origin, and the multiplicity of $(z-\zeta)^{1-\mu}$ is removed by requiring log $(z-\zeta)$ *to be real when $z-\zeta>0$.*

Definition 2. *The fractional derivative of order μ is defined, for a function $f(z)$, by*

$$D_z^{\mu}f(z)=\frac{1}{\Gamma(1-\mu)}\frac{d}{dz}\int_0^z\frac{f(\zeta)}{(z-\zeta)^{\mu}}d\zeta, \tag{8}$$

where $0 \leq \mu < 1$, $f(z)$ is an analytic function in a simply-connected region of the z-plane containing the origin, and the multiplicity of $(z-\zeta)^{-\mu}$ is removed as in Definition 1 above.

Definition 3. *Under the hypotheses of Definition 2, the fractional derivative of order $n+\mu$ is defined by*

$$D_z^{n+\mu} f(z) = \frac{d^n}{dz^n} D_z^{\mu} f(z), \tag{9}$$

where $0 \leq \mu < 1$ and $n \in \mathbb{N}_0 = \{0, 1, 2, \ldots\}$.

We begin by proving

Theorem 1. *If $f(z) \in \mathcal{B}_T(j, \lambda, \alpha)$, then*

$$\left|D_z^{-\mu} f(z)\right| \geq \frac{|z|^{1+\mu}}{\Gamma(2+\mu)} \left\{1 - \frac{(1-\alpha)\Gamma(j+2)\Gamma(2+\mu)}{\sigma(j+1,\alpha,\lambda)\Gamma(j+2+\mu)} |z|^j\right\} \tag{10}$$

and

$$\left|D_z^{-\mu} f(z)\right| \leq \frac{|z|^{1+\mu}}{\Gamma(2+\mu)} \left\{1 + \frac{(1-\alpha)\Gamma(j+2)\Gamma(2+\mu)}{\sigma(j+1,\alpha,\lambda)\Gamma(j+2+\mu)} |z|^j\right\}, \tag{11}$$

for $\mu > 0$ and $z \in \mathcal{U}$. The results (10) and (11) are sharp.

Proof . Define the function $G(z)$by

$$\begin{aligned} G(z) &= \Gamma(2+\mu) z^{-\mu} D_z^{-\mu} f(z) \\ &= z - \sum_{n=j+1}^{\infty} \frac{\Gamma(n+1)\Gamma(2+\mu)}{\Gamma(n+1+\mu)} a_n z^n \\ &= z - \sum_{n=j+1}^{\infty} \Psi(n) a_n z^n, \end{aligned}$$

where

$$\Psi(n) = \frac{\Gamma(n+1)\Gamma(2+\mu)}{\Gamma(n+1+\mu)} \qquad (n \geq j+1). \tag{12}$$

It easy to see that

$$0 < \Psi(n) \leq \Psi(j+1) = \frac{\Gamma(j+2)\Gamma(2+\mu)}{\Gamma(j+2+\mu)}. \tag{13}$$

Furthermore, it follows from Lemma 1 that

$$\sum_{n=j+1}^{\infty} a_n \leq \frac{1-\alpha}{\sigma(j+1,\alpha,\lambda)}, \tag{14}$$

Therefore, by using (13) and (14), we can see that

$$|G(z)| \geq |z| - \Psi(j+1)\,|z|^{j+1} \sum_{n=j+1}^{\infty} a_n \geq |z| - \frac{(1-\alpha)\Gamma(j+2)\Gamma(2+\mu)}{\sigma(j+1,\alpha,\lambda)\Gamma(j+2+\mu)}\,|z|^{j+1} \tag{15}$$

and

$$|G(z)| \leq |z| + \Psi(j+1)\,|z|^{j+1} \sum_{n=j+1}^{\infty} a_n \leq |z| + \frac{(1-\alpha)\Gamma(j+2)\Gamma(2+\mu)}{\sigma(j+1,\alpha,\lambda)\Gamma(j+2+\mu)}\,|z|^{j+1}, \tag{16}$$

which prove the inequalities of Theorem 1.

Finally, we can easily see that the results (10) and (11) are sharp for the function $f(z)$ given by

$$D_z^{-\mu} f(z) = \frac{z^{1+\mu}}{\Gamma(2+\mu)} \left\{ 1 - \frac{(1-\alpha)\Gamma(j+2)\Gamma(2+\mu)}{\sigma(j+1,\alpha,\lambda)\Gamma(j+2+\mu)} z^j \right\} \tag{17}$$

or

$$f(z) = z - \frac{1-\alpha}{\sigma(j+1,\alpha,\lambda)} z^{j+1}. \tag{18}$$

Theorem 2. *If* $f(z) \in \mathcal{B}_T(j,\lambda,\alpha)$, *then*

$$|D_z^{\mu} f(z)| \geq \frac{|z|^{1-\mu}}{\Gamma(2-\mu)} \left\{ 1 - \frac{(1-\alpha)\Gamma(j+2)\Gamma(2-\mu)}{\sigma(j+1,\alpha,\lambda)\Gamma(j+2-\mu)} |z|^j \right\} \tag{19}$$

and

$$|D_z^{\mu} f(z)| \leq \frac{|z|^{1-\mu}}{\Gamma(2-\mu)} \left\{ 1 + \frac{(1-\alpha)\Gamma(j+2)\Gamma(2-\mu)}{\sigma(j+1,\alpha,\lambda)\Gamma(j+2-\mu)} |z|^j \right\}, \tag{20}$$

for $0 \leq \mu < 1$ *and* $z \in \mathcal{U}$. *The results (19) and (20) are sharp.*

Proof. Define the function $H(z)$ by

$$\begin{aligned} H(z) &= \Gamma(2-\mu)z^{\mu}D_z^{\mu}f(z) \\ &= z-\sum_{n=j+1}^{\infty}\frac{\Gamma(n+1)\Gamma(2-\mu)}{\Gamma(n+1-\mu)}a_n z^n \\ &= z-\sum_{n=j+1}^{\infty}\Phi(n)a_n z^n, \end{aligned}$$

where

$$\Phi(n)=\frac{\Gamma(n)\Gamma(2-\mu)}{\Gamma(n+1-\mu)} \qquad (n\geq j+1). \tag{21}$$

It easy to see that

$$0<\Phi(n)\leq\Phi(j+1)=\frac{\Gamma(j+1)\Gamma(2-\mu)}{\Gamma(j+2-\mu)}. \tag{22}$$

Consequently, with the aid of (14) and (22), we have

$$|H(z)|\geq|z|-\Phi(j+1)\,|z|^{j+1}\sum_{n=j+1}^{\infty}na_n\geq|z|-\frac{(1-\alpha)\Gamma(j+2)\Gamma(2-\mu)}{\sigma(j+1,\alpha,\lambda)\Gamma(j+2-\mu)}\,|z|^{j+1} \tag{23}$$

and

$$|H(z)|\leq|z|+\Phi(j+1)\,|z|^{j+1}\sum_{n=j+1}^{\infty}na_n\leq|z|+\frac{(1-\alpha)\Gamma(j+2)\Gamma(2-\mu)}{\sigma(j+1,\alpha,\lambda)\Gamma(j+2-\mu)}\,|z|^{j+1}. \tag{24}$$

Now (19) and (20) follow from (23) and (24), respectively.

Finally, by taking the function $f(z)$ defined by

$$D_z^{\mu}f(z)=\frac{z^{1-\mu}}{\Gamma(2-\mu)}\left\{1-\frac{(1-\alpha)\Gamma(j+2)\Gamma(2-\mu)}{\sigma(j+1,\alpha,\lambda)\Gamma(j+2-\mu)}z^j\right\} \tag{25}$$

or for the function given by (18), the results (19) and (20) are easily seen to be sharp.

Remark 1. Letting $\mu=0$ in Theorem 1 and $\mu\longrightarrow1$ in Theorem 2, we shall obtain the corresponding results Theorem 3 and Theorem 4 in [3].

3. Fractional integral operator

We need the following definition of fractional integral operator given by Srivastava et al. [14].

Definition 4. *For real number $\eta > 0, \gamma$ and δ, the fractional integral operator $I_{0,z}^{\eta,\gamma,\delta}$ is defined by*

$$I_{0,z}^{\eta,\gamma,\delta} f(z) = \frac{z^{-\eta-\gamma}}{\Gamma(\eta)} \int_0^z (z-t)^{\eta-1} F(\eta+\gamma, -\delta; \eta; 1-t/z) f(t) dt, \tag{26}$$

where a function $f(z)$ is analytic in a simply-connected region of the z-plane containing the origin with the order

$$f(z) = O(|z|^{\varepsilon}) \qquad (z \longrightarrow 0),$$

with $\varepsilon > \max\{0, \gamma-\delta\} - 1$.

Here $F(a,b;c;z)$ is the Gauss hypergeometric function defined by

$$F(a,b;c;z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n (1)_n}, \tag{27}$$

where $(\nu)_n$ is the Pochhammer symbol defined by

$$(\nu)_n = \frac{\Gamma(\nu+k)}{\Gamma(\nu)} = \begin{cases} 1 & (n=0) \\ \nu(\nu+1)(\nu+2)\cdots(\nu+n-1) & (n \in \mathbb{N}) \end{cases} \tag{28}$$

and the multiplicity of $(z-t)^{\eta-1}$ is removed by requiring $\log(z-t)$ to be real when $z-t > 0$.

Remark 2. For $\gamma = -\eta$, we note that

$$I_{0,z}^{\eta,-\eta,\delta} f(z) = D_z^{-\eta} f(z).$$

In order to prove our result for the fractional integral operator, we have to recall here the following lemma due to Srivastava et al. [14].

Lemma 2. *If $\eta > 0$ and $n > \gamma - \delta - 1$, then*

$$I_{0,z}^{\eta,\gamma,\delta} z^n = \frac{\Gamma(n+1)\Gamma(n-\gamma+\delta+1)}{\Gamma(n-\gamma+1)\Gamma(n+\eta+\delta+1)} z^{n-\gamma}. \tag{29}$$

With aid of Lemma 2., we prove

Theorem 3. *Let $\eta > 0$, $\gamma > 2$, $\eta+\delta > -2$, $\gamma-\delta < 2, \gamma(\eta+\delta) \leq \eta(j+2)$, and $j \in \mathbb{N}$. If $f(z) \in \mathcal{B}_T(j,\lambda,\alpha)$, then*

$$\left|I_{0,z}^{\eta,\gamma,\delta}f(z)\right| \geq \frac{\Gamma(2-\gamma+\delta)\,|z|^{1-\gamma}}{\Gamma(2-\gamma)\Gamma(2+\eta+\delta)}\left\{1-\frac{(1-\alpha)(2-\gamma+\delta)_j(2)_j}{\sigma(j+1,\alpha,\lambda)(2-\gamma)_j(2-\gamma+\delta)_j}\,|z|^j\right\} \tag{30}$$

and

$$\left|I_{0,z}^{\eta,\gamma,\delta}f(z)\right| \leq \frac{\Gamma(2-\gamma+\delta)\,|z|^{1-\gamma}}{\Gamma(2-\gamma)\Gamma(2+\eta+\delta)}\left\{1+\frac{(1-\alpha)(2-\gamma+\delta)_j(2)_j}{\sigma(j+1,\alpha,\lambda)(2-\gamma)_j(2-\gamma+\delta)_j}\,|z|^j\right\} \tag{31}$$

for $z \in \mathcal{U}_0$, *where*

$$\mathcal{U}_0 = \begin{cases} \mathcal{U} & (\gamma \leq 1), \\ \mathcal{U}-\{0\} & (\gamma > 1). \end{cases} \tag{32}$$

The equalities in (30) and (31) are attained for the function $f(z)$ *given by (18).*

Proof . By using Lemma 2, we have

$$\begin{aligned} I_{0,z}^{\eta,\gamma,\delta}f(z) &= \frac{\Gamma(2-\gamma+\delta)}{\Gamma(2-\gamma)\Gamma(2+\eta+\delta)}z^{1-\gamma} \\ &= -\sum_{n=j+1}^{\infty}\frac{\Gamma(n+1)\Gamma(n-\gamma+\delta+1)}{\Gamma(n-\gamma+1)\Gamma(n+\eta+\delta+1)}a_n z^{n-\gamma} \qquad (z \in \mathcal{U}_0). \end{aligned}$$

Letting

$$\begin{aligned} \Omega(z) &= \frac{\Gamma(2-\gamma)\Gamma(2+\eta+\delta)}{\Gamma(2-\gamma+\delta)}z^{\gamma}I_{0,z}^{\eta,\gamma,\delta}f(z) \\ &= z-\sum_{n=j+1}^{\infty}\Delta(n)a_n z^n, \end{aligned} \tag{33}$$

where

$$\Delta(n) = \frac{(2-\gamma+\delta)_{n-1}(2)_{n-1}}{(2-\gamma)_{n-1}(2+\gamma+\delta)_{n-1}} \qquad (n \geq j+1), \tag{34}$$

we can see that the function $\Delta(n)$ is non-increasing for integers $n \geq j+1$, then we have

$$0 < \Delta(n) \leq \Delta(j+1) = \frac{(2-\gamma+\delta)_j(2)_j}{(2-\gamma)_j(2+\gamma+\delta)_j}. \tag{35}$$

Therefore, by using (14) and (35), we have

$$\begin{aligned} |\Omega(z)| &\geq |z| - \Delta(j+1)\,|z|^{j+1} \sum_{n=j+1}^{\infty} a_n \\ &\geq |z| - \frac{(1-\alpha)(2-\gamma+\delta)_j (2)_j}{\sigma(j+1,\alpha,\lambda)(2-\gamma)_j (2+\gamma+\delta)_j}\,|z|^{j+1} \end{aligned}$$

and

$$\begin{aligned} |\Omega(z)| &\leq |z| + \Delta(j+1)\,|z|^{j+1} \sum_{n=j+1}^{\infty} a_n \\ &\leq |z| + \frac{(1-\alpha)(2-\gamma+\delta)_j (2)_j}{\sigma(j+1,\alpha,\lambda)(2-\gamma)_j (2+\gamma+\delta)_j}\,|z|^{j+1} \end{aligned}$$

for $z \in \mathcal{U}_0$, where $\mathcal{U}_0$ is defined by (32). This completes the proof of Theorem 3.

Remark 3. Taking $\gamma = -\eta$ in Theorem 3, we get the result of Theorem 1.

4. Integral operators

Theorem 4. *Let the functions $f(z)$ defined by (3) be in the class $\mathcal{B}_T(j,\lambda,\alpha)$, and c be a real number such that $c > -1$. Then the function $F(z)$ defined by*

$$F(z) = \frac{c+1}{z^c} \int_0^z t^{c-1} f(t)dt \qquad (c > -1) \tag{36}$$

also belonging to the class $\mathcal{B}_T(j,\lambda,\alpha)$.

Proof. From (36) we have

$$F(z) = z - \sum_{n=j+1}^{\infty} \left(\frac{c+1}{c+n}\right) a_n z^n.$$

Therefore,

$$\sum_{n=j+1}^{\infty} \sigma(n,\alpha,\lambda) \left(\frac{c+1}{c+n}\right) a_n \leq \sum_{n=j+1}^{\infty} \sigma(n,\alpha,\lambda) a_n \leq 1-\alpha,$$

since $f(z) \in \mathcal{B}_T(j,\lambda,\alpha)$. Hence, by Lemma 1, $F(z) \in \mathcal{B}_T(j,\lambda,\alpha)$.

Theorem 5. *Let the function*

$$F(z) = z - \sum_{n=j+1}^{\infty} a_n z^n \qquad (a_n \geq 0)$$

be in the class $\mathcal{B}_T(j, \lambda, \alpha)$ and let c be a real number such that $c > -1$. Then the function given by (36) is univalent in $|z| < R^*$, where

$$R^* = R^*(n, \alpha, c) = \inf_n \left[\frac{\sigma(n, \alpha, \lambda)(c+1)}{n(1-\alpha)(c+n)} \right]^{1/(n-1)} \qquad (n \geq 2). \tag{37}$$

The result is sharp, with the function $f(z)$ given by

$$f(z) = z - \frac{(1-\alpha)(c+n)}{\sigma(n, \alpha, \lambda)(c+1)} z^n \qquad (n \geq 2). \tag{38}$$

Proof. From (36), we have

$$f(z) = \frac{z^{1-c}(z^c F(z))'}{c+1} = z - \sum_{n=j+1}^{\infty} \left(\frac{c+n}{c+1} \right) a_n z^n.$$

In order to obtain the required result, it suffices to show that
$|f'(z) - 1| < 1$ whenever $|z| < R^*$, where R^* is given by (37). Now

$$\left| f'(z) - 1 \right| \leq \sum_{n=j+1}^{\infty} \frac{n(c+n)}{c+1} a_n \left| z \right|^{n-1}.$$

Thus $|f'(z) - 1| < 1$ if

$$\sum_{n=j+1}^{\infty} \frac{n(c+n)}{c+1} a_n \left| z \right|^{n-1} < 1. \tag{39}$$

But from Lemma 1, (39) will be satisfied if

$$\frac{n(c+n)}{c+1} a_n \left| z \right|^{n-1} < \frac{\sigma(n, \alpha, \lambda)}{1-\alpha}, \tag{40}$$

that is, if

$$|z| \leq \left[\frac{\sigma(n, \alpha, \lambda)(c+1)}{n(1-\alpha)(c+n)} \right]^{1/(n-1)} \qquad (n \geq 2). \tag{41}$$

Therefore, $f(z)$ is univalent in $|z| < R^*$.

References

[1] M.K. Aouf, *On fractional derivatives and fractional integrals of certain subclasses of starlike and convex functions*, Math. Japon. 35 (1990), 831-837.

[2] A. Erdélyi, W. Magnus, F. Oberhettinger and F.G. Tricemi, *Tables of integral Transforms,* voll. II, McGraw-Hill Book Co., NewYork, Toronto and London,1954.

[3] B.A. Frasin, *On the analytic functions with negative coefficients*, Soochow J. Math.Vol 31, No.3 (2005), 349-359.

[4] V.P. Gupta and P.K. Jain, *Certain classes univalent analytic functions with negative coefficients II*, Bull. Austral. Math. Soc. 15 (1976), 467-473.

[5] K.B. Oldham and T. Spanier, *The Fractional Calculus: Theory and Applications of Differentiation and Integral to Arbitrary Order*, Academic Press, NewYork and London, 1974.

[6] S. Owa, *On the distortion theorems* I, Kyungpook Math. J. 18 (1978), 53-59.

[7] S. Owa, M. Saigo and H.M.Srivastava, *Some characterization theorems for starlike and convex functions involving a certain fractional integral operators*, J. Math. Anal. 140 (1989), 419-426.

[8] M. Saigo, *A remark on integral operators involving the Gauss hypergeometric functions*, Math. Rep. College General Ed. Kyushu Univ. 11 (1978),135-143.

[9] S.G. Samko, A.A. Kilbas and O.I. Marchev, *Integrals and Derivatives of Fractional Order and Some of Their Applications* (Russian), Nauka i Teknika, Minsk, 1987.

[10] H. Silverman, *Univalent functions with negative coefficients*, Proc. Amer. Math. Soc. 51 (1975),109-116.

[11] H.M.Srivastava and R.G. Buchman, *Convolution Integral Equations with Special Functions Kernels*, John Wiely and Sons, NewYork, London, Sydney and Toronto, 1977.

[12] H.M.Srivastava and S. Owa (Eds.), *Univalent functions, Fractional Calculus, and Their Applications*,Halsted Press (Ellis Horworod Limited, Chichester), John Wiely and Sons, NewYork, Chichester, Brisbane and Toronto, 1989.

[13] H.M.Srivastava, S. Owa and S.K. Chatterjea, *A note on certain classes of starlike functions*, Rend. Sem. Mat. Univ. Padova 77 (1987), 115-124.

[14] H.M.Srivastava, M. Saigo and S. Owa, *A class of distortion theorems involving certain operators of fractional calculus*, J. Math. Anal. Appl. 131 (1988), 412-420.

B.A. Frasin
Faculty of Science, Department of Mathematics
Al al-Bayt University, P.O. Box: 130095 Mafraq, Jordan
email:*bafrasin@yahoo.com*