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# PROPERTIES OF CERTAIN CLASS OF P-VALENT MEROMORPHIC FUNCTIONS ASSOCIATED WITH NEW INTEGRAL OPERATOR

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Abstract. In this paper, we investigated some interesting properties of certain class of p-valent meromorphic functions which is defined by new integral operator.

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## 1. Introduction

Let $\Sigma_{p,n}$denote the class of functions of the form:

$$f(z)=\frac{1}{z^p}+\sum_{k=n}^{\infty}a_kz^k \qquad (p\in\mathbb{N}=\{1,2,...\};n>-p\}), \tag{1.1}$$

which are analytic and p-valent in the punctured open unit disk $U^*=\{z:z\in\mathbb{C}$ and $0<|z|<1\}=U\backslash\{0\}$.

For two functions $f$ and $g$ analytic in $U$, we say that $f$ is subordinate to $g$, written symbolically as $f(z)\prec g(z)$, if there exists a Schwarz function $w$, which (by definition) is analytic in $U$ with $w(0)=0$ and $|w(z)|<1$ $(z\in U)$ such that $f(z)=g(w(z))$. In particular, if the function $g$ is univalent in $U$, then we have the following equivalence (see [8, p.4]):

$$f(z)\prec g(z)\Leftrightarrow f(0)=g(0) \textit{ and } f(U)\subset g(U).$$

Let $\varphi(r,s;z):\mathbb{C}^2\times U\rightarrow\mathbb{C}$ and $h(z)$ be univalent in $U$. If $p(z)$ is analytic in $U$ and satisfies the first order differential subordination:

$$\varphi(p(z),zp^{'}(z);z)\prec h(z) \tag{1.2}$$

then $p(z)$ is a solution of the differential subordination (1.2). The univalent function $q(z)$ is called a dominant of the solutions if $p(z)\prec q(z)$ for all $p(z)$ satisfying (1.2).

A univalent dominant $\widetilde{q}$ that satisfies $\widetilde{q} \prec q$ for all dominants of (1.2) is called the best dominant (see [8]).
For two functions $f_j(z) \in \Sigma_{p,n}$ $(j = 1, 2)$, given by

$$f_j(z) = \frac{1}{z^p} + \sum_{k=n}^{\infty} a_{k,j} z^k \quad (j = 1, 2), \tag{1.3}$$

we define the Hadamard product (or convolution) of $f_1(z)$ and $f_2(z)$ by

$$(f_1 * f_2)(z) = \frac{1}{z^p} + \sum_{k=n}^{\infty} a_{k,1} a_{k,2} z^k = (f_2 * f_1)(z).$$

For $\ell > 0, \lambda \geq 0$ and $m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$, El-Ashwah [5] defined the multiplier transformations $J_p^m(\lambda, \ell)$ of functions $f \in \Sigma_{p,n}$ by

$$J_p^m(\lambda, \ell) f(z) = \frac{1}{z^p} + \sum_{k=n}^{\infty} \left( \frac{\ell + \lambda(k+p)}{\ell} \right)^m a_k z^k \quad (\ell > 0; \lambda \geq 0; z \in U^*). \tag{1.4}$$

Obviously, we have

$$J_p^{m_1}(\lambda, \ell)(J_p^{m_2}(\lambda, \ell) f(z)) = J_p^{m_1+m_2}(\lambda, \ell) f(z) = J_p^{m_2}(\lambda, \ell)(J_p^{m_1}(\lambda, \ell) f(z)), \tag{1.5}$$

for all integers $m_1$ and $m_2$.

We note that
(i) $J_1^m(1, \ell) f(z) = I(m, \ell) f(z)$ (see Cho et al. [3, 4]);
(ii) $J_1^m(\lambda, 1) f(z) = D_{\lambda,p}^m f(z)$ (see Al-Oboudi and Al-Zkeri [1]);
(iii) $J_1^m(1, 1) f(z) = I^m f(z)$ (see Uralegaddi and Somanatha [9]).

Now, we define the integral operator $\mathcal{L}_p^m(\lambda, \ell) f(z)$ $(\lambda, \ell > 0)$ as follows:

$$\begin{aligned}
\mathcal{L}_p^0(\lambda, \ell) f(z) &= f(z), \\
\mathcal{L}_p^1(\lambda, \ell) f(z) &= \left(\tfrac{\ell}{\lambda}\right) z^{-p-\left(\frac{\ell}{\lambda}\right)} \int_0^z t^{\left(\frac{\ell}{\lambda}+p-1\right)} f(t) dt \quad (f \in \Sigma_{p,n}; z \in U^*), \\
\mathcal{L}_p^2(\lambda, \ell) f(z) &= \left(\tfrac{\ell}{\lambda}\right) z^{-p-\left(\frac{\ell}{\lambda}\right)} \int_0^z t^{\left(\frac{\ell}{\lambda}+p-1\right)} \mathcal{L}_p^1(\lambda, \ell) f(t) dt \quad (f \in \Sigma_{p,n}; z \in U^*),
\end{aligned}$$

and, in general,

$$\mathcal{L}_p^m(\lambda,\ell)f(z) = \left(\tfrac{\ell}{\lambda}\right) z^{-p-\left(\frac{\ell}{\lambda}\right)} \int_0^z t^{\left(\frac{\ell}{\lambda}+p-1\right)} \mathcal{L}_p^{m-1}(\lambda,\ell)f(t)dt$$

$$= \underbrace{\mathcal{L}_p^1(\lambda,\ell)\left(\tfrac{1}{z^p(1-z)}\right) * \mathcal{L}_p^1(\lambda,\ell)\left(\tfrac{1}{z^p(1-z)}\right) * \ldots * \mathcal{L}_p^1(\lambda,\ell)\left(\tfrac{1}{z^p(1-z)}\right)}_{m-times} * f(z)$$

$$(f \in \Sigma_{p,n}; m \in \mathbb{N}_0; p \in \mathbb{N}; z \in U^*). \tag{1.6}$$

We note that if $f(z) \in \sum_{p,n}$, then from (1.1) and (1.6), we have

$$\mathcal{L}_p^m(\lambda,\ell)f(z) = \frac{1}{z^p} + \sum_{k=n}^{\infty} \left[\frac{\ell}{\ell+\lambda(k+p)}\right]^m a_k z^k$$

$$(\ell > 0; \lambda \geq 0; p \in \mathbb{N}; m \in \mathbb{N}_0; z \in U^*). \tag{1.7}$$

From (1.7), it is easy verify that

$$\lambda z(\mathcal{L}_p^{m+1}(\lambda,\ell)f(z))' = \ell\mathcal{L}_p^m(\lambda,\ell)f(z) - (\ell+p\lambda)\mathcal{L}_p^{m+1}(\lambda,\ell)f(z) \quad (\lambda > 0). \tag{1.8}$$

We note that:
$(i)$ $\mathcal{L}_p^{\alpha}(1,1)f(z) = P_p^{\alpha}f(z)$ (see Aqlan et al. [2]);
$(ii)$ $\mathcal{L}_1^{\alpha}(1,\beta)f(z) = P_{\beta}^{\alpha}f(z)$ (see Lashin [7]).
Also we note that
$(i)$ $\mathcal{L}_p^m(1,\ell)f(z) = \mathcal{L}_{p,\ell}^m f(z)$, where $\mathcal{L}_{p,\ell}^m f(z) = \frac{1}{z^p} + \sum_{k=n}^{\infty}\left(\frac{\ell}{\ell+k+p}\right)^m a_k z^k$;
$(ii)$ $\mathcal{L}_p^m(\lambda,1)f(z) = \mathcal{L}_{p,\lambda}^m f(z)$, where $\mathcal{L}_{p,\lambda}^m f(z) = \frac{1}{z^p} + \sum_{k=0}^{\infty}\left(\frac{1}{1+\lambda(k+p)}\right)^m a_k z^k$;
$(iii)$ $\mathcal{L}_p^m(1,1)f(z) = \mathcal{L}_p^m f(z)$, where $\mathcal{L}_{p,\lambda}^m f(z) = \frac{1}{z^p} + \sum_{k=0}^{\infty}\left(\frac{1}{k+p+1}\right)^m a_k z^k$.

## 2. Main results

Unless otherwise mentioned we shall assume throughout the paper that $\lambda, \ell > 0, p \in \mathbb{N}, m \in \mathbb{N}_0$ and $-1 \leq B < A \leq 1$.

To prove our results we need the following lemma.

**Lemma 1 [6].** *Let $h(z)$ be analytic and convex (univalent) in $U$, $h(0) = 1$, and let*

$$\varphi(z) = 1 + c_{p+n} z^{p+n} + ... \tag{2.1}$$

*be analytic in $U$. If*

$$\varphi(z) + \frac{1}{\delta} z\varphi'(z) \prec h(z),$$

*then for $\delta \neq 0$ and $\operatorname{Re}\delta \geq 0$*

$$\varphi(z) \prec \psi(z) = \left(\tfrac{\delta}{p+n}\right) z^{-\left(\frac{\delta}{p+n}\right)} \int_0^z t^{\left(\frac{\delta}{p+n}\right)-1} h(t)dt \qquad (z \in U) \tag{2.2}$$

*and $\psi(z)$ is the best dominant of (2.2).*

**Theorem 1**. *If $f(z) \in \sum_{p,n}$ and $0 < \gamma < 1$. Suppose that*

$$\sum_{k=n}^{\infty} c_k |a_k| \leq 1, \tag{2.3}$$

*where*

$$c_k = \frac{1-B}{A-B} . \frac{\ell^m[\ell + (1-\gamma)\lambda(k+p)]}{[\ell + \lambda(k+p)]^{m+1}}. \tag{2.4}$$

(i) *If $-1 \leq B \leq 0$, then*

$$(1-\gamma)z^p \mathcal{L}_p^m(\lambda, \ell)f(z) + \gamma z^p \mathcal{L}_p^{m+1}(\lambda, \ell)f(z) \prec \frac{1+Az}{1+Bz}, \tag{2.5}$$

(ii) *If $-1 \leq B \leq 0$ and $\rho \geq 1$, then for $z \in U$*

$$\operatorname{Re}\left\{ \left(z^p \mathcal{L}_p^m(\lambda, \ell)f(z)\right)^{\frac{1}{\rho}} \right\} > \left\{ \left( \frac{\ell}{\lambda(1-\gamma)(p+n)} \right) \int_0^1 t^{\left(\frac{\ell}{\lambda(1-\gamma)(p+n)}\right)-1} \left( \frac{1-At}{1-Bt} \right) dt \right\}^{\frac{1}{\rho}}. \tag{2.6}$$

*The result is sharp.*

*Proof.* (i) Let

$$G(z) = (1-\gamma)z^p \mathcal{L}_p^m(\lambda, \ell)f(z) + \gamma z^p \mathcal{L}_p^{m+1}(\lambda, \ell)f(z), \tag{2.7}$$

then

$$G(z) = 1 + \sum_{k=n}^{\infty} \frac{\ell^m[\ell + (1-\gamma)\lambda(k+p)]}{[\ell + \lambda(k+p)]^{m+1}} a_k z^{k+p}. \tag{2.8}$$

Using (2.3) for $-1 \leq B \leq 0$ and $z \in U$, we have

$$\begin{aligned} \left| \frac{G(z)-1}{A - BG(z)} \right| &= \left| \frac{\sum_{k=n}^{\infty} \frac{\ell^m[\ell+(1-\gamma)\lambda(k+p)]}{[\ell+\lambda(k+p)]^{m+1}} a_k z^{k+p}}{A - B - B\sum_{k=n}^{\infty} \frac{\ell^m[\ell+(1-\gamma)\lambda(k+p)]}{[\ell+\lambda(k+p)]^{m+1}} a_k z^{k+p}} \right| \\ &\leq \frac{\sum_{k=n}^{\infty} c_k |a_k|}{1 - B + B\sum_{k=n}^{\infty} c_k |a_k|} \\ &\leq 1, \end{aligned}$$

which proves (i) of Theorem 1.

(ii) Put

$$\varphi(z) = z^p J_p^{m+1}(\lambda, \ell) f(z). \tag{2.9}$$

Then the function $\varphi(z)$ take the form (2.1) and analytic in $U$. Differentiating (2.9) with respect to $z$ and using (1.8), we obtain

$$\begin{aligned} (1-\gamma) z^p \mathcal{L}_p^m(\lambda,\ell) f(z) + \gamma z^p \mathcal{L}_p^{m+1}(\lambda,\ell) f(z) &= \varphi(z) + \frac{(1-\gamma)\lambda}{\ell} z\varphi'(z) \\ &\prec \frac{1+Az}{1+Bz}. \end{aligned} \tag{2.10}$$

Application of Lemma 1 gives

$$\varphi(z) \prec \left(\tfrac{\ell}{\lambda(1-\gamma)(p+n)}\right) z^{-\left(\frac{\ell}{\lambda(1-\gamma)(p+n)}\right)} \int_0^z t^{\left(\frac{\ell}{\lambda(1-\gamma)(p+n)}\right)-1} \left(\frac{1+At}{1+Bt}\right) dt$$

which is equivalent to,

$$z^p \mathcal{L}_p^{m+1}(\lambda,\ell) f(z) = \left(\tfrac{\ell}{\lambda(1-\gamma)(p+n)}\right) \int_0^1 u^{\left(\frac{\ell}{\lambda(1-\gamma)(p+n)}\right)-1} \left(\frac{1+Auw(z)}{1+Buw(z)}\right) du, \tag{2.11}$$

where $w(z)$ is analytic in $U$ with $w(0) = 0$ and $|w(z)| < 1$ $(z \in U)$.
It follows from (2.11) that

$$\operatorname{Re}\left\{ z^p \mathcal{L}_p^{m+1}(\lambda,\ell) f(z) \right\} > \left(\tfrac{\ell}{\lambda(1-\gamma)(p+n)}\right) \int_0^1 u^{\left(\frac{\ell}{\lambda(1-\gamma)(p+n)}\right)-1} \left(\frac{1-Au}{1-Bu}\right) du > 0 \; (z \in U).$$

Therefore, with the elementary inequality $\text{Re}(w^{\frac{1}{\rho}}) \geq (\text{Re}(w))^{\frac{1}{\rho}}$ for $\text{Re}(w) > 0$ and $\rho \in \mathbb{N}$, the inequality (2.6) follows immediately.

To show the sharpness of (2.6), we take $f(z) \in \sum_{p,n}$ defined by

$$z^p \mathcal{L}_p^{m+1}(\lambda, \ell) f(z) = \left(\tfrac{\ell}{\lambda(1-\gamma)(p+n)}\right) \int_0^1 u^{\left(\frac{\ell}{\lambda(1-\gamma)(p+n)}\right)-1} \left(\frac{1+Auz^n}{1+Buz^n}\right) du. \tag{2.12}$$

For this function we find that

$$(1-\gamma) z^p \mathcal{L}_p^m(\lambda, \ell) f(z) + \gamma z^p \mathcal{L}_p^{m+1}(\lambda, \ell) f(z) = \frac{1+Az^n}{1+Bz^n},$$

and

$$z^p \mathcal{L}_p^{m+1}(\lambda, \ell) f(z) \longrightarrow \left(\tfrac{\ell}{\lambda(1-\gamma)(p+n)}\right) \int_0^1 u^{\left(\frac{\ell}{\lambda(1-\gamma)(p+n)}\right)-1} \left(\frac{1-Au}{1-Bu}\right) du \quad as\ z \longrightarrow e^{\frac{i\pi}{n}}.$$

Hence the proof of Theorem 1 is complete.

**Theorem 2.** *Let $f(z) \in \sum_{p,n}$ be given by (1.1) and*

$$c_k \geq \begin{cases} 1, & k = n, n+1, ..., q \\ c_{q+1}, & k = q+1, q+2, ..., \end{cases}$$

*where $c_k$ is given by (2.4) and satisfying the condition (2.3), define the partial sums $s_1(z)$ and $s_q(z)$ as follows:*

$$s_1(z) = z^{-p}$$

*and*

$$s_q(z) = z^{-p} + \sum_{k=n}^{q} |a_k| z^k \ \ (q \in \mathbb{N}; q > n), \tag{2.13}$$

*then we have*

$$(i)\ \text{Re}\left\{\frac{f(z)}{s_q(z)}\right\} > 1 - \frac{1}{c_{q+1}} \quad (z \in U; q \in \mathbb{N}, q > n), \tag{2.14}$$

*and*

$$(ii)\ \text{Re}\left\{\frac{s_q(z)}{f(z)}\right\} > 1 - \frac{1}{1+c_{q+1}} \quad (z \in U; q \in \mathbb{N}, q > n). \tag{2.15}$$

*The estimates in (2.14) and (2.15) are sharp for $q \in \mathbb{N}, q > n$.*

*Proof.* (i) Under the hypothesis of Theorem 2, we can see from (2.3) that

$$\sum_{k=n}^{q} |a_k| + c_{q+1} \sum_{k=q+1}^{\infty} |a_k| \leq \sum_{k=n}^{\infty} c_k |a_k| \leq 1, \tag{2.16}$$

By setting

$$g_1(z) = c_{q+1}\left\{\frac{f(z)}{s_q(z)} - (1 - \frac{1}{c_{q+1}})\right\}$$

$$= 1 + \frac{c_{q+1}\sum\limits_{k=q+1}^{\infty} a_k z^{k+p}}{1 + \sum\limits_{k=n}^{q} a_k z^{k+p}}, \tag{2.17}$$

and applying (2.16), we find that

$$\left|\frac{g_1(z) - 1}{g_1(z) + 1}\right| \leq \frac{c_{q+1}\sum\limits_{k=q+1}^{\infty} |a_k|}{2 - 2\sum\limits_{k=n}^{q} |a_k| - c_{q+1}\sum\limits_{k=q+1}^{\infty} |a_k|} \leq 1 \qquad (z \in U), \tag{2.18}$$

which readily yields the assertion (2.14) of Theorem 2 If we take

$$f(z) = z^{-p} + \frac{z^{q+1}}{c_{q+1}}, \tag{2.19}$$

with $z = re^{\overline{\frac{i\pi}{q+p+1}}}$ and let $r \to 1^-$, we obtain

$$\frac{f(z)}{s_q(z)} = 1 + \frac{z^{q+p+1}}{c_{q+1}} \to 1 - \frac{1}{c_{q+1}} \quad ,$$

which shows that the bound in (2.14) is best possible for each $q \in \mathbb{N}, q > n$.

(ii) Similarly, if we put

$$g_2(z) \; = \; (1 + c_{q+1})\left(\frac{s_q(z)}{f(z)} - \frac{c_{q+1}}{1 + c_{q+1}}\right)$$

$$= 1 - \frac{(1 + c_{q+1})\sum\limits_{k=q+1}^{\infty} |a_k|\, z^{k+p}}{1 + \sum\limits_{k=n}^{\infty} |a_k|\, z^{k+p}},$$

and make use of (2.16), we can deduce that

$$\left|\frac{g_2(z) - 1}{g_2(z) + 1}\right| \leq \frac{(1 + c_{q+1})\sum\limits_{k=q+1}^{\infty} |a_k|}{2 - 2\sum\limits_{k=n}^{q} |a_k| + (1 - c_{q+1})\sum\limits_{k=q+1}^{\infty} |a_k|} \leq 1 \qquad (z \in U), \tag{2.20}$$

which yields inequality (2.15) of Theorem 2. The bound in (2.15) is sharp for each $q \in \mathbb{N}, q > n$, with the extremal function $f(z)$ given by (2.19). The proof of Theorem 2 is now complete.

**Theorem 3.** *Let $f(z) \in \sum_{p,n}$ be given by (1.1) and*

$$c_k \geq \begin{cases} (k/p)\ , & \text{if } k = n, n+1, ..., q, \\ \dfrac{c_{q+1}}{q+1}(k/p)\,, & \text{if } k = q+1, q+2, ... \ . \end{cases}$$

*where $c_k$ is given by (2.4) and satisfying the condition (2.3), then we have*

$$(i)\,\mathrm{Re}\left\{\frac{f'(z)}{s'_q(z)}\right\} > 1 - \frac{q+1}{c_{q+1}} \quad (z \in U; q \in \mathbb{N}, q > n), \tag{2.21}$$

*and*

$$(ii)\,\mathrm{Re}\left\{\frac{s'_q(z)}{f'(z)}\right\} > 1 - \frac{q+1}{q+1+c_{q+1}} \quad (z \in U;\ r \in \mathbb{N}, q > n). \tag{2.22}$$

*The estimates in* (2.21) *and* (2.22) *are sharp for $q \in \mathbb{N}, q > n$.*
*The results are sharp with the function $f(z)$ given by (2.19).*

*Proof.* By setting

$$g(z) = \frac{c_{q+1}}{q+1}\left\{\frac{f'(z)}{s'_q(z)} - \left(1 - \frac{q+1}{c_{q+1}}\right)\right\}$$

$$= \frac{1 + \dfrac{c_{q+1}}{q+1}\displaystyle\sum_{k=q+1}^{\infty}(k/p)\,a_k z^{k+p} + \sum_{k=n}^{q}(k/p)\,a_k z^{k+p}}{1 + \displaystyle\sum_{k=n}^{q}(k/p)\,a_k z^{k+p}}. \tag{2.23}$$

Then we have

$$\left|\frac{g(z)-1}{g(z)+1}\right| \leq \frac{\dfrac{c_{q+1}}{q+1}\displaystyle\sum_{k=q+1}^{\infty}(k/p)\,|a_k|}{2 - 2\displaystyle\sum_{k=n}^{q}(k/p)\,|a_k| - \dfrac{c_{q+1}}{q+1}\displaystyle\sum_{k=q+1}^{\infty}(k/p)\,|a_k|}. \tag{2.24}$$

Now

$$\left|\frac{g(z)-1}{g(z)+1}\right| \leq 1,$$

if

$$\sum_{k=2}^{q}(k/p)\,|a_k| + \frac{c_{q+1}}{q+1}\sum_{k=q+1}^{\infty}(k/p)\,|a_k| \leq 1, \tag{2.25}$$

since the left hand side of (2.25) is bounded above by $\sum\limits_{k=2}^{\infty} c_k\,|a_k|$ if

$$\sum_{k=n}^{q}(c_k - (k/p))\,|a_k| + \sum_{k=q+1}^{\infty}\left(c_k - \frac{c_{q+1}}{q+1}\,(k/p)\right)|a_k| \geq 0 \tag{2.26}$$

and the proof of (2.21) is completed.

To prove the result (2.22), we define the function $h(z)$ by

$$h(z) = \left(\frac{q+1+c_{q+1}}{q+1}\right)\left\{\frac{s_q'(z)}{f'(z)} - \frac{c_{q+1}}{q+1+c_{q+1}}\right\}$$

$$= 1 - \frac{\left(1+\dfrac{c_{q+1}}{q+1}\right)\sum\limits_{k=q+1}^{\infty}(k/p)\,a_k z^{k+p}}{1+\sum\limits_{k=n}^{\infty}(k/p)\,a_k z^{k+p}},$$

and making use of (2.26), we deduce that

$$\left|\frac{h(z)-1}{h(z)+1}\right| \leq \frac{\left(1+\dfrac{c_{q+1}}{q+1}\right)\sum\limits_{k=q+1}^{\infty}(k/p)\,|a_k|}{2-2\sum\limits_{k=n}^{q}(k/p)\,|a_k| - \left(1+\dfrac{c_{q+1}}{q+1}\right)\sum\limits_{k=q+1}^{\infty}(k/p)\,|a_k|} \leq 1,$$

which leads us immediately to the assertion (2.22) of Theorem 3.

**Remark.** By specializing the parameters $p, \lambda, \ell$ and $m$ we obtain various results for different operators.

## References

[1] F. M. Al-Oboudi and H. A. Al-Zkeri, *Applications of Briot-Bouquet differential subordination to certain classes of meromorphic functions*, Arab J. Math Sci., 12(2005), no. 1, 1-14.

[2] E. Aqlan, J. M. Jahangiri and S. R. Kulkarni, *Certain integral operators applied to meromorphic p-valent functions*, J. Nat. Geom., 24(2003), 111-120.

[3] N. E. Cho, O. S. Kwon and H. M Srivastava, *Inclusion and argument properties for certain subclasses of meromorphic functions associated with a family of multiplier transformations*, J. Math. Anal. Appl., 300(2004), 505-520.

[4] N. E. Cho, O. S. Kwon and H. M. Srivastava, *Inclusion relationships for certain subclasses of meromorphic functions associated with a family of multiplier transformations*, Integral Transforms Spec. Funct., 16(2005), no. 18, 647-659.

[5] R. M. El-Ashwah, *A note on certain meromorphic p-valent functions*, Appl. Math. Letters, 22 (2009), 1756-1759.

[6] D. J. Hallenbeck and St. Ruscheweyh, *Subordinations by convex functions*, Proc. Amer. Math. Soc. 52(1975), 191-195.

[7] A. Y. Lashin, *On certain subclass of meromorphic functions associated with certain integral operators*, Comput. Math Appl., 59(2010), no.1, 524-531.

[8] S. S. Miller and P. T. Mocanu, *Differential subordinations : Theory and Applications*, Series on Monographs and Textbooks in Pure and Appl. Math. No.225 Marcel Dekker, Inc., New York, 2000.

[9] B. A. Uralegaddi and C. Somanatha, *New criteria for meromorphic starlike univalent functions*, Bull. Austral. Math. Soc.43 (1991), 137-140.

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