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THE MATRIX TRANSFORMATIONS ON ORLICZ SPACE OF χ

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Abstract. Let χ denote the space of all gai sequences and Λ the space of all analytic sequences. First we show that the set $E = \left\{ s^{(k)} : k = 1, 2, 3, \cdots \right\}$ is a determining set for χ_M. The set of all finite matrices transforming χ_M into FK-space Y denoted by $(\chi_M : Y)$. We characterize the classes $(\chi_M : Y)$ when $Y = (c_0)_\pi, c_\pi, \chi_M, \ell_\pi, \ell_s, \Lambda_\pi, h_\pi$. In summary we have the following table:

$\nearrow$	$(c_0)_\pi$	c_π	χ_M	ℓ_π	ℓ_s	Λ_π	h_π
χ_M	Necessary and sufficient condition on the matrix are obtained						

But the approach to obtain these result in the present paper is by determining set for χ_M. First, we investigate a determining set for χ_M and then we characterize the classes of matrix transformations involving χ_M and other known sequence spaces.

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1. Introduction

A complex sequence, whose k^{th} terms is x_k is denoted by $\{x_k\}$ or simply x. Let w be the set of all sequences $x = (x_k)$ and ϕ be the set of all finite sequences. Let ℓ_∞, c, c_0 be the sequence spaces of bounded, convergent and null sequences $x = (x_k)$ respectively. In respect of ℓ_∞, c, c_0 we have
$\|x\| = \overset{sup}{k} |x_k|$, where $x = (x_k) \in c_0 \subset c \subset \ell_\infty$. A sequence $x = \{x_k\}$ is said to be analytic if $sup_k |x_k|^{1/k} < \infty$. The vector space of all analytic sequences will be denoted by Λ. A sequence x is called entire sequence if $lim_{k\to\infty} |x_k|^{1/k} = 0$. The vector space of all entire sequences will be denoted by Γ.χ was discussed in Kamthan [19]. Matrix transformation involving χ were characterized by Sridhar [20] and Sirajiudeen [21]. Let χ be the set of all those sequences $x = (x_k)$ such that $(k! |x_k|)^{1/k} \to 0$ as $k \to \infty$. Then χ is a metric space with the metric

$$d(x, y) = sup_k \left\{ (k! |x_k - y_k|)^{1/k} : k = 1, 2, 3, \cdots \right\}$$

Orlicz [4] used the idea of Orlicz function to construct the space (L^M). Lindenstrauss and Tzafriri [5] investigated Orlicz sequence spaces in more detail, and they proved that every Orlicz sequence space ℓ_M contains a subspace isomorphic to $\ell_p (1 \leq p < \infty)$. Subsequently different classes of sequence spaces defined by Parashar and Choudhary[6], Mursaleen et al.[7], Bektas and Altin[8], Tripathy et al.[9], Rao and subramanian[10] and many others. The Orlicz sequence spaces are the special cases of Orlicz spaces studied in Ref[11].
Recall([4],[11]) an Orlicz function is a function $M : [0, \infty) \to [o, \infty)$ which is continuous, non-decreasing and convex with $M(0) = 0, M(x) > 0$, for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$. If convexity of Orlicz function M is replaced by $M(x+y) \leq M(x)+M(y)$ then this function is called modulus function, introduced by Nakano[18] and further discussed by Ruckle[12] and Maddox[13] and many others.
An Orlicz function M is said to satisfy Δ_2- condition for all values of u, if there exists a constant $K > 0$, such that $M(2u) \leq KM(u)(u \geq 0)$. The Δ_2- condition is equivalent to $M(\ell u) \leq K\ell M(u)$, for all values of u and for $\ell > 1$. Lindenstrauss and Tzafriri[5] used the idea of Orlicz function to construct Orlicz sequence space

$$\ell_M = \left\{ x \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty, \, for some \, \rho > 0 \right\}. \tag{1}$$

The space ℓ_M with the norm

$$\|x\| = inf \left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1 \right\} \tag{2}$$

becomes a Banach space which is called an Orlicz sequence space. For $M(t) = t^p, 1 \leq p < \infty$, the space ℓ_M coincide with the classical sequence space ℓ_p. Given a sequence $x = \{x_k\}$ its n^{th} section is the sequence $x^{(n)} = \{x_1, x_2, ..., x_n, 0, 0, ...\}$ $\delta^{(n)} = (0, 0, ..., 1, 0, 0, ...)$, 1 in the n^{th} place and zero's else where; and $s^{(k)} = (0, 0, ..., 1, -1, 0, ...)$, 1 in the n^{th} place,-1 in the $(n+1)^{th}$place and zero's else where.
An FK-space (Frechet coordinate space) is a Frechet space which is made up of numerical sequences and has the property that the coordinate functionals $p_k(x) = x_k \, (k = 1, 2, 3, \ldots)$ are continuous. We recall the following definitions [see [15]].
An FK-space is a locally convex Frechet space which is made up of sequences and has the property that coordinate projections are continuous. An metric-space (X, d) is said to have AK (or sectional convergence) if and only if $d\left(x^{(n)}, x\right) \to x$ as $n \to \infty$.[see[15]] The space is said to have AD (or) be an AD space if ϕ is dense in X. We note that AK implies AD by [14].
If X is a sequence space, we define

(i)$X^{'}$ = the continuous dual of X.
(ii)$X^{\alpha} = \{a = (a_k) : \sum_{k=1}^{\infty} |a_k x_k| < \infty, \, for each \, x \in X\}$;
(iii)$X^{\beta} = \{a = (a_k) : \sum_{k=1}^{\infty} a_k x_k \, is \, convergent, \, for each \, x \in X\}$;
(iv)$X^{\gamma} = \left\{a = (a_k) : \overset{sup}{n} \left|\sum_{k=1}^{n} a_k x_k\right| < \infty, \, for each \, x \in X\right\}$;
(v)Let X be an FK-space$\supset \phi$. Then $X^{f} = \left\{f(\delta^{(n)}) : f \in X^{'}\right\}$.
$X^{\alpha}, X^{\beta}, X^{\gamma}$ are called the $\alpha-$(or Kö the-T öeplitz)dual of X, $\beta-$ (or generalized Kö the-T öeplitz)dual of X, $\gamma-$dual of X. Note that $X^{\alpha} \subset X^{\beta} \subset X^{\gamma}$. If $X \subset Y$ then $Y^{\mu} \subset X^{\mu}$, for $\mu = \alpha, \beta, or \, \gamma$.
Lemma 1. $(See \, (15, Theorem 7.27))$. Let X be an FK-space $\supset \phi$. Then
(i)$X^{\gamma} \subset X^{f}$. (ii)If X has AK, $X^{\beta} = X^{f}$. (iii)If X has AD, $X^{\beta} = X^{\gamma}$.

2.Definitions and Prelimiaries

Let w denote the set of all complex double sequences $x = (x_k)_{k=1}^{\infty}$ and $M : [0, \infty) \to [0, \infty)$ be an Orlicz function, or a modulus function. Let
$\chi_M = \left\{x \in w : lim_{k \to \infty} \left(M\left(\frac{(k!|x_k|)^{1/k}}{\rho}\right)\right) = 0 \, for some \, \rho > 0\right)$,
$\Gamma_M = \left\{x \in w : lim_{k \to \infty} \left(M\left(\frac{|x_k|^{1/k}}{\rho}\right)\right) = 0 \, for some \, \rho > 0\right)$ and
$\Lambda_M = \left\{x \in w : sup_k \left(M\left(\frac{|x_k|^{1/k}}{\rho}\right)\right) < \infty \, for some \, \rho > 0\right)$
The space χ_M is a metric space with the metric

$$d\,(x,\,y) = inf\left\{\rho > 0 : sup_k \left(M\left(\frac{(k!\,|x_k - y_k|)^{1/k}}{\rho}\right)\right) \leq 1\right\} \tag{3}$$

The space Γ_M and Λ_M is a metric space with the metric

$$d\,(x,\,y) = inf\left\{\rho > 0 : sup_k \left(M\left(\frac{|x_k - y_k|^{1/k}}{\rho}\right)\right) \leq 1\right\} \tag{4}$$

Let ℓ_s denote the space of all those complex sequences $x = \{x_k\}$ such that $\{x_1, x_1 + x_2, x_1 + x_2 + x_3, \cdots x_1 + x_2 + \cdots + x_k + \cdots\}$ belongs to ℓ with the norm $\|x\|_s = |x_1| + |x_1 + x_2| + \cdots + |x_1 + x_2 + \cdots + x_k| + \cdots$,

$\Gamma_{\pi} = \left\{x = \{x_k\} : \left(\frac{x_k}{\pi_k}\right) \in \Gamma\right\}$ and $\Lambda_{\pi} = \left\{x = \{x_k\} : \left(\frac{x_k}{\pi_k}\right) \in \Lambda\right\}$.
Then Γ_{π} and Λ_{π} are FK-spaces with the metric $d\,(x, y) = sup_k \left\{\left|\frac{x_k - y_k}{\pi_k}\right|^{1/k} : k = 1, 2, 3, \cdots\right\}$.
$h_{\pi} = \left\{x = \{x_k\} : \left(\frac{x_k}{\pi_k}\right) \in h\right\}$. Then h_{π} is a BK-space with the norm $\|x\| = \sum_{k=1}^{\infty} k \left|\frac{x_k}{\pi_k} - \frac{x_{k+1}}{\pi_{k+1}}\right|$.

$(\ell_\infty)_\pi = m_\pi = \left\{x = \{x_k\} \in w : \left(\frac{x_k}{\pi_k}\right) \in m\right\}$, $(c_0)_\pi = \left\{x = \{x_k\} \in w : \left(\frac{x_k}{\pi_k}\right) \in c_0\right\}$, $(c)_\pi = \left\{x = \{x_k\} \in w : \left(\frac{x_k}{\pi_k}\right) \in c\right\}$. In respect of $m_\pi, (c_0)_\pi, c_\pi$ are BK-spaces with the norm $\|x\|_\pi = sup_k \left|\frac{x_k}{\pi_k}\right|$,

$\ell_\pi = \left\{x = \{x_k\} \in w : \left(\frac{x_k}{\pi_k}\right) \in \ell\right\}$, ℓ_π is a BK-space with the norm $\|x\| = \sum_{k=1}^{\infty} \left|\frac{x_k}{\pi_k}\right|$. We call $(c_0)_\pi, c_\pi, \ell_\pi, \Lambda_\pi, h_\pi$ are rate spaces. [See [24]]

Let X be an BK-space. Then $D = D(X) = \{x \in \phi : \|x\| \leq 1\}$ we do not assumethat $X \supset \phi$ (i.e)$D = \phi \bigcap (unit\, closed\, sphere\, in\, X)$

Let X be an BK space. A subset E of ϕ will be called a determining set for X if $D(X)$ is the absolutely convexhull of E. In respect of a metric space (X, d), $D = \{x \in \phi : d(x, 0) \leq 1\}$.

Given a sequence $x = \{x_k\}$ and an infinite matrix $A = (a_{nk})$, $n, k = 1, 2, \cdots$ then $A-$ transform of x is the sequence $y = (y_n)$ where $y_n = \sum_{k=1}^{\infty} a_{nk} x_k \, (n, k = 1, 2, \cdots)$. Whenever $\sum a_{nk} x_k$ exists.

Let X and Y be FK-spaces. If $y \in Y$ whenever $x \in X$, then the class of all matrices A is denoted by $(X : Y)$.

Lemma 2. Let X be a FK-space and E is determining set for X. Let Y be an FK-space and A is a infinite matrix. Suppose that either X has AK or A is row finite. Then $A \in (X : Y)$ if and only if (1) The columns of A belong to Y and (2) $A[E]$ is a bounded subset of Y.

3.Main Results

Proposition 1. χ_M has AK, where M is a modulus function.

Proof: Let $x = \{x_k\} \in \chi_M$, but then $\left\{M\left(\frac{(k!|x_k|)^{1/k}}{\rho}\right)\right\} \in \chi$, and hence $sup_{k \geq n+1}\left(M\left(\frac{(k!|x_k|)^{1/k}}{\rho}\right)\right) \to$ as $n \to \infty$. Therefore

$$d\left(x, x^{[n]}\right) = inf\left\{\rho > 0 : sup_{k \geq n+1}\left(M\left(\frac{(k!\,|x_k|)^{1/k}}{\rho}\right)\right) \leq 1\right\} \to 0 \, as \, n \to \infty \quad (5)$$

$\Rightarrow x^{[n]} \to x$ as $n \to \infty$, implying that χ_M has AK. This completes the proof.

Proposition 2. Let $\left\{s^{(k)} : k = 1, 2, 3, \cdots\right\}$ be the set of all sequences in ϕ each of whose non-zero terms $\pm$. Let $E = \left\{s^{(k)} : k = 1, 2, 3, \cdots\right\}$ then E is a determining set for the space χ_M.

Proof: Step 1: Recall that χ_M is a metric space with the metric

$$d\left(x,\,y\right)=inf\left\{\rho>0:sup_k\left(M\left(\frac{(k!|x_k-y_k|)^{1/k}}{\rho}\right)\right)\leq 1\right\}$$

Let A be the absolutely convex hull of E. Let $x\in A$. Then $x=\sum_{k=1}^{m}t_ks^{(k)}$ with

$$\sum_{k=1}^{m}|t_k|\leq 1\,and\,s^{(k)}\in E \tag{6}$$

Then $d\left(x,0\right)\leq|t_1|\,d\left(s^{(1)},0\right)+|t_2|\,d\left(s^{(2)},0\right)+\cdots+|t_1|\,d\left(s^{(m)},0\right)$. But $d\left(s^{(k)},0\right)=1$ for $k=1,2,3,\cdots,m$. Hence $d\left(x,0\right)\leq\sum_{k=1}^{m}|t_k|\leq 1$ by using (6). Also $x\in\phi$. Hence $x\in D$. Thus we have

$$A\subset D \tag{7}$$

step 2: Let $x\in D$.

$\Rightarrow x\in\phi$ and $d\left(x,0\right)\leq 1$.

$\Rightarrow x=\{x_1,x_2,\cdots,x_m\}$ and

$$sup\left\{M\left(\frac{\left(1!\,|x_1|\right)^{1/1}}{\rho}\right),M\left(\frac{\left(2!\,|x_2|\right)^{1/2}}{\rho}\right),\cdots,M\left(\frac{\left(m!\,|x_m|\right)^{1/m}}{\rho}\right),\right\}\leq 1 \tag{8}$$

Case 1. Suppose that

$$M\left(\frac{(1!|x_1|)^{1/1}}{\rho}\right)\geq M\left(\frac{(2!|x_2|)^{1/1}}{\rho}\right)\geq\cdots\geq M\left(\frac{(m!|x_m|)^{1/m}}{\rho}\right)$$

Let $\epsilon_i=sgn\left(M\left(\frac{(i!x_i)}{\rho}\right)\right)=\frac{M\left(\frac{(i!|x_i|)}{\rho}\right)}{\left(M\left(\frac{(i!x_i)}{\rho}\right)\right)}$ for $i=1,2,3,\cdots,m$.

Take $S_j=\{\epsilon_2,\epsilon_2,\cdots,\epsilon_j,0,0,\cdots\}$ for $j=1,2,3,\cdots,m$.

Then $S_j\in E$ for $j=1,2,3,\cdots,m$. Also

$x=\left(M\left(\frac{(1!|x_1|)^{1/1}}{\rho}\right)-M\left(\frac{(2!|x_2|)^{1/2}}{\rho}\right)\right)s_1+\left(M\left(\frac{(2!|x_2|)^{1/2}}{\rho}\right)-M\left(\frac{(3!|x_3|)^{1/3}}{\rho}\right)\right)s_2+\cdots+\left(M\left(\frac{(m!|x_m|)^{1/m}}{\rho}\right)-M\left(\frac{((m+1)!|x_{m+1}|)^{1/m+1}}{\rho}\right)\right)s_m=t_1s_1+t_2s_2+\cdots+t_ms_m$.

So that

$t_1 + t_2 + \cdots + t_m = M\left(\frac{(1!|x_1|)^{1/1}}{\rho}\right) - M\left(\frac{((m+1)!|x_{m+1}|)^{1/m+1}}{\rho}\right) = M\left(\frac{(1!|x_1|)^{1/1}}{\rho}\right)$ because $M\left(\frac{((m+1)!|x_{m+1}|)^{1/m+1}}{\rho}\right) = .0$
Therefore $t_1 + t_2 + \cdots + t_m \leq 1$ by using (8). Hence $x \in A$. Thus we have $D \subset A$.

Case (ii): Let y be x and let $M\left(\frac{(1!|y_1|)}{\rho}\right) \geq M\left(\frac{(2!|y_2|)}{\rho}\right) \geq \cdots \geq M\left(\frac{(m!|y_m|)}{\rho}\right)$

Express y as a member of A as in case(i). Since E is invariant under permutation of the terms of its members, so is A. Hence $x \in A$. Thus $D \subset A$. Therefore in both cases

$$D \subset A \tag{9}$$

From (7) and (9) $A = D$. Consequently E is a determining set for χ_M. This completes the proof.
Proposition 3. An infinite matrix $A = (a_{nk})$ is in the class

$$A \in (\chi_M : (c_0)_\pi) \Leftrightarrow lim_{n\to\infty}\left(\frac{a_{nk}}{\pi_n}\right) = 0 \tag{10}$$

$$\Leftrightarrow sup_{nk}\left|\frac{a_{n1} + \cdots + a_{nk}}{\pi_n}\right| < \infty. \tag{11}$$

Proof: In Lemma 3. Take $X = \chi_M$ has AK property take $Y = (c_0)_\pi$ be an FK-space. Further more χ_M is a determining set E (as in given Proposition 4.2). Also $A[E] = A\left(s^{(k)}\right) = \{(a_{n1} + a_{n2} + \cdots)\}$. Again by Lemma 3. $A \in (\chi_M : (c_0)_\pi)$ if and only if (i)The columns of A belong to $(c_0)_\pi$ and (ii)$A\left(s^{(k)}\right)$ is a bounded subset $(c_0)_\pi$. But the condition
(i)$\Leftrightarrow \left\{\frac{a_{nk}}{\pi_n} : n = 1, 2, \cdots\right\}$ is exists for all k.
(ii)$\Leftrightarrow sup_{n,k}\left|\frac{a_{n1}+\cdots+a_{nk}}{\pi_n}\right| < \infty$.
Hence we conclude that $A \in (\chi_M : (c_0)_\pi) \Leftrightarrow$ conditions (10) and (11) are satisfied. This is completes the proof.
Omitting the proofs, we formulate the following results:
Proposition 4. An infinite matrix $A = (a_{nk})$ is in the class

$$A \in (\chi_M : c_\pi) \Leftrightarrow lim_{n\to\infty}\left(\frac{a_{nk}}{\pi_n}\right) exists (k = 1, 2, 3, ...) \tag{12}$$

$$\Leftrightarrow sup_{nk}\left|\frac{a_{n1} + \cdots + a_{nk}}{\pi_n}\right| < \infty. \tag{13}$$

Proposition 5. An infinite matrix $A = (a_{nk})$ is in the class

$$A \in (\chi_M : \chi_M) \Leftrightarrow sup_{nk}\left(M\left(\frac{\left(n!\,|a_{n1}+\cdots+a_{nk}|^{1/n}\right)}{\rho}\right)\right) < \infty. \tag{14}$$

$$\Leftrightarrow lim_{n\to\infty}\left(M\left(\frac{(n!\,|a_{nk}|)^{1/n}}{\rho}\right)\right) = 0, for \quad k = 1,2,3,... \tag{15}$$

$$\Leftrightarrow d\left(a_{n1}, a_{n2}, \cdots, a_{nk}\right) is \quad bounded \tag{16}$$

for each metric d on χ_M and for all n, k.

Proposition 6. An infinite matrix $A = (a_{nk})$ is in the class

$$A \in (\chi_M : \ell_\pi) \Leftrightarrow \sum_{n=1}^{\infty} |a_{nk}|\, converges(k = 1,2,3,...) \tag{17}$$

$$\Leftrightarrow sup_{nk}\sum_{n=1}^{\infty}\left|\frac{a_{nk}}{\pi_n}\right| < \infty \tag{18}$$

Propositio n 7. An infinite matrix $A = (a_{nk})$ is in the class

$$A \in (\chi_M : \ell_s) \Leftrightarrow sup_k \sum_{n=1}^{\infty} |a_{nk}| < \infty \tag{19}$$

Proposition 8. An infinite matrix $A = (a_{nk})$ is in the class

$$A \in (\chi_M : \Lambda_\pi) \Leftrightarrow sup_{nk}\left(\left|\sum_{\gamma=1}^{k}\frac{a_{n\gamma}}{\pi_n}\right|^{1/n}\right) < \infty \tag{20}$$

$$\Leftrightarrow d\left(a_{n1}, a_{n2}, \cdots a_{nk}\right) is\ bounded \tag{21}$$

for each metric d on Λ_π and for all n, k.

Proposition 9. An infinite matrix $A = (a_{nk})$ is in the class

$$A \in (\chi_M : h_\pi) \Leftrightarrow \left\{\frac{a_{nk}}{\pi_n} : n = 1,2,3,\cdots\right\}\ is\ exists\ for\ each\ k. \tag{22}$$

$$\Leftrightarrow sup_k \sum_{n=1}^{\infty}\left|\frac{a_{n1}+a_{n2}+\cdots+a_{nk}}{\pi_n} - \frac{a_{n+1,1}+a_{n+2,2}+\cdots+a_{n+1,k}}{\pi_{n+1}}\right| < \infty \tag{23}$$

References

[1] A.K.Snyder and A.Wilansky, Inclusion Theorems and semi conservative FK-spaces, *Rocky Mountain Journal of Math.*, **2**(1972), 595-603.

[2] K.Chandrasekhara Rao and T.G.Srinivasalu, The Hahn sequence space-II, *Journal of Faculty of Education*, **1(2)**(1996), 43-45.

[3] A.K.Snyder, Consistency theory in semi conservative spaces, *Studia Math.*, **5**(1982), 1-13.

[4] W.Orlicz, *Ü*ber Raume $\left(L^M\right)$ *Bull. Int. Acad. Polon. Sci. A*, **(**1936), 93-107.

[5] J.Lindenstrauss and L.Tzafriri, On Orlicz sequence spaces, *Israel J. Math.*, **10** (1971), 379-390.

[6] S.D.Parashar and B.Choudhary, Sequence spaces defined by Orlicz functions, *Indian J. Pure Appl. Math.* , **25(4)**(1994), 419-428.

[7] M.Mursaleen,M.A.Khan and Qamaruddin, Difference sequence spaces defined by Orlicz functions, *Demonstratio Math.* , **Vol. XXXII** (1999), 145-150.

[8] C.Bektas and Y.Altin, The sequence space $\ell_M(p,q,s)$ on seminormed spaces, *Indian J. Pure Appl. Math.*, **34(4)** (2003), 529-534.

[9] B.C.Tripathy,M.Etand Y.Altin, Generalized difference sequence spaces defined by Orlicz function in a locally convex space, *J. Analysis and Applications*, **1(3)**(2003), 175-192.

[10] K.Chandrasekhara Rao and N.Subramanian, The Orlicz space of entire sequences, *Int. J. Math. Math. Sci.*, **68**(2004), 3755-3764.

[11] M.A.Krasnoselskii and Y.B.Rutickii, Convex functions and Orlicz spaces, *Gorningen, Netherlands*, **1961**.

[12] W.H.Ruckle, FK spaces in which the sequence of coordinate vectors is bounded, *Canad. J. Math.*, **25**(1973), 973-978.

[13] I.J.Maddox, Sequence spaces defined by a modulus, *Math. Proc. Cambridge Philos. Soc*, **100(1)** (1986), 161-166.

[14] H.I.Brown, The summability field of a perfect $\ell - \ell$ method of summation, *J. Anal. Math.*, **20**(1967), 281-287.

[15] A.Wilansky, Summability through Functional Analysis, *North-Holland Mathematical Studies, North-Holland Publishing, Amsterdam*, **Vol.85**(1984).

[16] P.K.Kamthan and M.Gupta, Sequence spaces and series. Lecture Notes in Pure and Applied Mathematics, *Marcel Dekker Inc. New York*, **65**(1981).

[17] K.Chandrasekhara Rao and N.Subramanian, Semi analytic spaces, *Science Letters*, **26**(2003), 146-149.

[18] Nakano, Concave modulars, *J. Math. Soc. Japan*, **5**(1953), 29-49.

[19] P.K.Kamthan, Bases in a certain class of Frechet space, *Tamkang J. Math.*, **1976**, 41-49.

[20] S.Sridhar, A matrix transformation between some sequence spaces, *Acta Ciencia Indica*, **5** (1979), 194-197.

[21] S.M.Sirajiudeen, Matrix transformations of $c_0(p), \ell_\infty(p), \ell^p$ and ℓ into χ, *Indian J. Pure Appl. Math.*, **12(9)** (1981), 1106-1113.

[22] K.Chandrasekhara Rao and N.Subramanian, The semi Orlicz space of analytic sequences, *Thai Journal of Mathematics*, **Vol.2 (No.1)**, (2004), 197-201.

[23] N.Subramanian, B.C.Tripathy and C.Murugesan, The semi Orlicz space of $cs \bigcap d_1$, *Communications of the Korean Mathematical Society*, **in press**.

[24] E.Jurimae, Properties of domains of matrix mappings on rate-spaces and spaces with speed, *Acta et Commentationes Universitatis Tartuensis de Mathematica*, **No. 970**, (1994), 53-64.

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