GENERATING THE SET OF NONDOMINATED POINTS USING MCRIT APPLICATION

by
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Abstract. The reality itself is multicriterial for this the problem of choosing an alternative which could be considered acceptable in respect with many points of view often contradictory characterize the most of the human activities . Unlike the case of unicriterial optimization a solution of multiobjective problem is more a concept then definition. Practically it isn't one global solution and frequently is necessary to determinate a set of alternatives appreciated reasonable in respect with the imposed criterions. This study insists on presenting the applicability of the multicriterial programming in the economic activity. Because the choosing of the final alternative is based on generating the set of nondominated points algorithm I created the software application package MCRIT (developed in C++) which allow us to generate the Nex set and also the choosing of final alternative using the method of ideal solution. The program was tested on small, medium and large systems and it allow us to obtain the set of nondominated points in a reasonable amount of time.
This paper presents a case study which was solved with MCRIT package. Because the Nex set is to large I propose to limit it by imposing an acceptable threshold (an acceptable distance by the ideal solution) to that the variants are accepted by the decision maker. So became efficiently the use of ELECTRE method to determine the final solution. Moreover, besides the fact that the decision maker is moving in the space of reasonable solutions, it also could considerate the qualitative criterions which accompanying the reality of model and which couldn't be surprised by the mathematical programming.

Keywords: multicriterial programming, nondominated points, MCRIT.

Further I propose a case study that organizes of plant breeding system using the mathematical method of the multicriterial decisions. Because the multicriterial point of view with respect to generating the set of nondominated points constitute the chosen methodology for designing the agricultural production, it imposed the necessity of creating an IT application to allow us the finding of the solutions in a reasonable time.
Many of the existing programs are based on the weighting method. This way imposes large costs and a lack of mobility for the decision maker. So, analyzing the alternatives I decided to design an original IT solution which is based on the nondominated points theory. This is developed in C++ (the MCRIT application).

The application was tested on small, medium and large models. Obviously in case of the large models the searching time of the nondominated points is increasing considerably and in this case we must have a big computation unit or a net of units.
For determining the final solution is proposed the SIDEAL IT application based on ideal solution method. For the ELECTRE method wasn't necessary to design a IT solution because there are many good solution on the market already.
In the case study proposed below the solutions are generated using MCRIT.
The goal of this example is to underline some aspects of multicriterial decision problems. The accent is on the way of finding the best alternative from the feasible solutions set of an multiobjective problem.
As a reference model we'll consider a 9040ha field and we further the research on the farms specialized in vegetal productions. In this methodology proposed for the economical optimization, the both components the breeding structure and the production technologies were considered simultaneously. The model is composed by 4 objective functions, 18 variables (representing the areas which will be occupied by the technological variants) and it has 15 restrictions.
From the objective functions was selected those who modeling the following aspects: the minimizing of the consumed energy (FO_1); the maximizing the amount of the produced energy (FO_2); the maximizing of the net production value (FO_3); the maximizing of the brut benefits
(FO_4).

Result the mathematical model:

$$\min \quad FO_1(x) = \sum_{j=1}^{18} C_{1j} \cdot x_j \qquad \max \quad FO_2(x) = \sum_{j=1}^{18} C_{2j} \cdot x_j$$

$$\max \quad FO_3(x) = \sum_{j=1}^{18} C_{3j} \cdot x_j \qquad \max \quad FO_4(x) = \sum_{j=1}^{18} C_{4j} \cdot x_j$$

$$\sum_{j=1}^{18} A_{1j} \cdot x_j \le 35000000 \qquad \sum_{j=1}^{18} A_{2j} \cdot x_j \le 28500000$$

$$\sum_{j=1}^{18} A_{3j} \cdot x_j \le 1500 \qquad \sum_{j=1}^{18} A_{4j} \cdot x_j \le 650$$

$$\sum_{j=1}^{18} A_{5j} \cdot x_j \le 6500000 \qquad \sum_{j=1}^{18} A_{6j} \cdot x_j \le 75000$$

$$\sum_{j=1}^{18} A_{7j} \cdot x_j \le 16000 \qquad \sum_{j=1}^{18} A_{8j} \cdot x_j \le 950000$$

$$\sum_{j=1}^{18} x_j = 9040 \qquad \sum_{j=1}^{6} x_j \le 3600$$

$$\sum_{j=7}^{12} x_j \le 2000 \;, \qquad \sum_{j=13}^{18} x_j \le 5200$$

$$\sum_{j=1}^{6} A_{9\,j} \cdot x_j \ge 6100 \qquad \sum_{j=7}^{12} A_{10\,j} \cdot x_j \ge 2100$$

$$\sum_{j=13}^{18} A_{11\,j} \cdot x_j \ge 2500 \qquad x_j \ge 0 ,\; j = \overline{1,18} \,.$$

The data is recorded in following table:

Restrictions	SYMBOL	Wheat						Barley			
		fertilizer N_{109}/P_{117}unerb.*	Fertilizer N_{109}/P_{117}erb.*	Fertilizer N_{82}/P_{53}unerb.*	Fertilizer N_{82}/P_{53}erb.*	Fertilizer N_{64}/P_{48}unerb.*	Fertilizer N_{64}/P_{48}erb.*	Fertilizer N_{119}/P_{138}unerb.*	Fertilizer N_{119}/P_{138}erb.*	Fertilizer N_{90}/P_{71}unerb.*	Fertilizer N_{90}/P_{71}erb.*
min. consumed energy kwh	FO_1	6534	6663	5431	5554	4921	5045	6885	7018	5719	5838
max. produced energy, kwh	FO_2	34090	34090	32530	32530	31674	31674	43204	43204	40953	40953
max.val. net prod. , *1000 lei	FO_3	1607	1487	2017	1899	1984	1984	1056	936	1498	1380
max. brut benefit , *1000 lei	FO_4	1407	1411	1831	1836	1798	1802	911	916	1363	1369
respecting the total budget, *1000 lei	R_1	3461	3456	2823	2817	2725	2721	3556	3552	2871	2866
respecting the material budget, *1000 lei	R_2	3052	3172	2437	2555	2345	2465	3126	3246	2466	2584
fertilizer .with nitrogen, t/ha.	R_3	0.109	0.109	0.182	0.182	0.064	0.064	0.119	0.119	0.09	0.09

Restrictions	SYMBOL	Wheat						Barley			
		fertilizer $N_{109/}/P_{117}$unerb.*	Fertilizer $N_{109/}/P_{117}$erb.*	Fertilizer $N_{82/}/P_{53}$unerb.*	Fertilizer $N_{82/}/P_{53}$erb.*	Fertilizer $N_{64/}/P_{48}$unerb.*	Fertilizer $N_{64/}/P_{48}$erb.*	Fertilizer $N_{119/}/P_{138}$unerb.*	Fertilizer $N_{119/}/P_{138}$erb.*	Fertilizer $N_{90/}/P_{71}$unerb.*	Fertilizer $N_{90/}/P_{71}$erb.*
fertilizer .with phosphorus, t/ha.	R_4	0.117	0.117	0.053	0.053	0.048	0.048	0.138	0.138	0.071	0.071
respecting the salary budget, *1000 lei	R_5	408	285	387	263	380	256	430	306	406	282
respecting the manual man power day. hour	R_6	4.56	2.46	4.32	2.22	4.24	2.14	4.77	2.67	4.5	2.4
respecting the mechanized man power day. hour	R_7	1.29	1.36	1.23	1.3	1.2	1.27	1.24	1.54	1.3	1.37
respecting the fuel stock	R_8	80.2	84.2	76.2	80	75	78.9	77.6	81.3	73.5	77.1
respecting the arable areas ha	R_9	1	1	1	1	1	1	1	1	1	1
max. wheat area ha	R_{10}	1	1	1	1	1	1				
max. barley area, ha	R_{11}							1	1	1	1
max. corn area ha	R_{12}										
min. wheat amount, t	R_{13}	4.613	4.613	4.41	4.41	4.286	4.286				
min. barley amount, t	R_{14}							5.163	5.163	4.894	4.894
min. corn amount, t	R_{15}										

*The harvest was done by the combine
**The harvest was done manually.

Restrictions	**SYMBOL**	**Barley**		**Corn**						**SIGN**	**free terms**
		fertilizer N_{64}/P_{48}unerb.*	fertilizer N_{64}/P_{48}erb.*	fertilizer N_{363}/P_{220}unerb.*	fertilizer N_{363}/P_{220}erb.total*	fertilizer N_{363}/P_{220}erb/row*	fertilizer N_{182}/P_{68}unerb.*	fertilizer N_{182}/P_{68}erb.total**	fertilizer N_{182}/P_{68}erb/row*		
min. consumed energy kwh	FO_1	4830	4949	13808	14178	14088	8050	7903	8200	=	MIN
max. produced energy, kwh	FO_2	38041	38041	71161	71161	71161	62931	62931	62931	=	MAX
max.val. net prod. , *1000 lei	FO_3	1479	1359	861	455	827	2160	2281	2212	=	MAX
max. brut benefit , *1000 lei	FO_4	1342	1347	629	780	148	1916	1835	2459	=	MAX
respecting the total budget, *1000 lei	R_1	2591	2587	7077	6933	6565	4894	4973	4392	$\leq$	$35*10^6$
respecting the material budget, *1000 lei	R_2	2203	2323	5754	6164	5791	3684	3562	3732	$\leq$	$28.5*10^6$
fertilizer .with nitrogen, t/ha.	R_3	0.064	0.064	0.363	0.363	0.363	0.182	0.182	0.182	$\leq$	1500
fertilizer .with phosphorus, t/ha.	R_4	0.048	0.048	0.22	0.22	0.22	0.068	0.068	0.068	$\leq$	650
respecting the salary budget, *1000 lei	R_5	388	364	1323	769	773	1210	1411	660	$\leq$	$6.5*10^6$
respecting the manual man power day. hour	R_6	4.31	2.21	17.07	8.23	8.18	15.77	21.77	6.87	$\leq$	75000
respecting the mechanized man power day. hour	R_7	1.24	1.31	2.7	2.6	2.7	2.4	1.7	2.4	$\leq$	16000
respecting the fuel stock	R_8	70	79.6	168.3	165	169.4	147.4	102.3	148.5	$\leq$	950000
respecting the arable areas ha	R_9	1	1	1	1	1	1	1	1	=	9040
max. wheat area ha	R_{10}									$\leq$	3600
max. barley area, ha	R_{11}	1	1							$\leq$	2000
max. corn area ha	R_{12}			1	1	1	1	1	1	$\leq$	5200
min. wheat amount, t	R_{13}									$\geq$	6100
min. barley amount, t	R_{14}	4.546	4.546							$\geq$	2100
min. corn amount, t	R_{15}			8.073	8.073	8.073	7.127	7.127	7.127	$\geq$	20500

*The harvest was done by the combine
**The harvest was done manually.

The initial structure was V^0:

Specification		**Results**	**Observations**
The values of the objectives functions	min. consumed energy kwh	5.9413*10^7	
	max. produced energy, kwh	41.2824*10^7	
	max. val. net prod. , *1000 lei	1.65411*10^7	
	max. brut benefit , *1000 lei	1.50215*10^7	
The structure of breeding and production technology	Wheat :X_1-X_6,ha	X_1=1684 X_2=336 X_3=1580	= 3600
	Barley: X_7-X_{12},ha	X_{12}=2000	=2000
	Corn: X_{13}-X_{18} ,ha	X_{16}=3440	<5200
Necessary resource	total budget, *1000 lei	33459240	-1540760
	material budget, *1000 lei	27374780	-1125220
	fertilizer .with nitrogen, t/ha.	1261.82	-238.18
	fertilizer .with phosphorus, t/ha.	650	0
	salary budget, *1000 lei	6284692	-215308
	manual man power day. hour	74000	-1000
	mechanized man power day. hour	15449.72	-551.28
	gas l	950000	0
The quantities which can be obtained	Wheat amount [t]	16286.1	+10186.1
	Barley amount [t]	9092	+6992
	Corn amount [t]	27771.1	+7271.1

Obvious our interest is if this solution is less than another.
The study points to underline the opportunity of using the multicriterial programming, this imposing reasonable solutions with respects to all four criterions. In this case it could be preferred the weighting method but it must be underline the fact that besides of the incoherence of the mode of forming the acceptable weighting , the cost of applying for this method could be high if it is generated a large set of optimal solutions. For that I propose the solving of the problem in discussion using a binary relation in the feasible set P which gives us the preferences of the decision maker face with the diverse possible alternatives in respects with considered criterions.
We'll say that $y \in P$ is dominated by the $x \in P$ if $F(x) \geq F(y)$ and exists i=1,p such as $F_i(x)>F_i(y)$, where $F(x)=(F_1(x),...,F_p(x))$ – the vector of considered criterions. We'll say that $x^* \in P$ is nondominated point if it isn't another $x \in P$ which can dominate it.
So a final solution must be chosen from the N set of nondominated points.
Respecting the above statements we obtain 41 nondominated points (from roughly 12400 possible solutions) dates which are organized in the following table where we

registered the nondominated solution and the values of the objective functions, further calculating the distance from the ideal solution.

V_i	FO_1	FO_2	FO_3	FO_4	$\sum\lvert z^* - z\rvert$
V_1	$6.28396*10^7$	$44.3359*10^7$	$1.83435*10^7$	$1.76145*10^7$	$2.33127*10^7$
V_2	$6.09697*10^7$	$44.0886*10^7$	$1.88548*10^7$	$\mathbf{1.81338*10^7}$	$2.33282*10^7$
V_3	$6.29713*10^7$	$\mathbf{45.824*10^7}$	$1.77887*10^7$	$1.7335*10^7$	$0.98407*10^7$
V_4	$\mathbf{5.50358*10^7}$	$40.6591*10^7$	$1.7837*10^7$	$1.64648*10^7$	$5.43761*10^7$
V_5	$6.02303*10^7$	$44.2233*10^7$	$1.85046*10^7$	$1.79156*10^7$	$2.14292*10^7$
V_6	$6.15791*10^7$	$44.2712*10^7$	$1.87781*10^7$	$1.808*10^7$	$2.22421*10^7$
V_7	$6.14661*10^7$	$44.215*10^7$	$1.88399*10^7$	$1.81096*10^7$	$2.25997*10^7$
V_8	$6.15644*10^7$	$44.2597*10^7$	$1.88605*10^7$	$1.80792*10^7$	$2.22608*10^7$
V_9	$6.16773*10^7$	$44.3157*10^7$	$1.8801*10^7$	$1.80496*10^7$	$2.19028*10^7$
V_{10}	$6.10586*10^7$	$44.1269*10^7$	$1.88708*10^7$	$1.81092*10^7$	$2.30427*10^7$
V_{11}	$6.22134*10^7$	$45.3773*10^7$	$1.82213*10^7$	$1.75787*10^7$	$1.28736*10^7$
V_{12}	$6.2059*10^7$	$44.9887*10^7$	$1.8438*10^7$	$1.77642*10^7$	$1.62029*10^7$
V_{13}	$5.97877*10^7$	$42.0724*10^7$	$1.80363*10^7$	$1.80373*10^7$	$4.32232*10^7$
V_{14}	$6.25724*10^7$	$45.598*10^7$	$1.80181*10^7$	$1.74682*10^7$	$1.13392*10^7$
V_{15}	$6.23425*10^7$	$45.1629*10^7$	$1.82775*10^7$	$1.7677*10^7$	$1.49921*10^7$
V_{16}	$5.97876*10^7$	$41.8772*10^7$	$1.77545*10^7$	$1.7951*10^7$	$4.55432*10^7$
V_{17}	$6.07154*10^7$	$45.1049*10^7$	$1.82037*10^7$	$1.75957*10^7$	$1.41*10^7$
V_{18}	$5.97822*10^7$	$42.8808*10^7$	$1.84451*10^7$	1.8113710^7	$3.46505*10^7$
V_{19}	$6.10092*10^7$	$44.4996*10^7$	$1.86064*10^7$	$1.79281*10^7$	$1.97118*10^7$
V_{20}	$5.8486*10^7$	$41.7549*10^7$	$1.80604*10^7$	$1.8088*10^7$	$4.50217*10^7$
V_{21}	$6.20018*10^7$	$44.4683*10^7$	$1.85399*10^7$	$1.79575*10^7$	$2.10545*10^7$
V_{22}	$6.29596*10^7$	$45.8235*10^7$	$1.77941*10^7$	$1.73407*10^7$	$0.98229*10^7$
V_{23}	$6.2806*10^7$	$45.4329*10^7$	$1.80094*10^7$	$1.75243*10^7$	$1.31764*10^7$

V_{24}	$6.10976*10^7$	$45.3275*10^7$	$1.79827*10^7$	$1.74699*10^7$	$1.26031*10^7$
V_{25}	$6.15328*10^7$	$44.8046*10^7$	$1.83035*10^7$	$1.77557*10^7$	$1.76607*10^7$
V_{26}	$6.11541*10^7$	$45.3067*10^7$	$1.81539*10^7$	$1.75256*10^7$	$1.26407*10^7$
V_{27}	$6.00818*10^7$	$44.0758*10^7$	$1.87798*10^7$	$1.80641*10^7$	$2.2713*10^7$
V_{28}	$6.1546*10^7$	$45.5296*10^7$	$1.79527*10^7$	$1.74167*10^7$	$1.11137*10^7$
V_{29}	$5.97022*10^7$	$45.0374*10^7$	$1.81393*10^7$	$1.75449*10^7$	$1.38781*10^7$
V_{30}	$5.95626*10^7$	$43.9526*10^7$	$1.87912*10^7$	$1.80847*10^7$	$2.33938*10^7$
V_{31}	$5.55836*10^7$	$40.6591*10^7$	$1.77097*10^7$	$1.76158*10^7$	$5.39002*10^7$
V_{32}	$5.68845*10^7$	$41.8992*10^7$	$1.78001*10^7$	$1.7878*10^7$	$4.24475*10^7$
V_{33}	$5.68354*10^7$	$41.5837*10^7$	$1.7982*10^7$	$1.80307*10^7$	$4.52188*10^7$
V_{34}	$5.96045*10^7$	$44.8009*10^7$	$1.79024*10^7$	$1.73207*10^7$	$1.66049*10^7$
V_{35}	$5.62916*10^7$	$41.0189*10^7$	$1.7593*10^7$	$1.77865*10^7$	$5.09562*10^7$
V_{36}	$6.15745*10^7$	$45.5294*10^7$	$1.81909*10^7$	$1.73935*10^7$	$1.09292*10^7$
V_{37}	$6.06662*10^7$	$44.3267*10^7$	**$1.88951*10^7$**	$1.78984*10^7$	$2.08408*10^7$
V_{38}	$6.19533*10^7$	$45.7454*10^7$	$1.79886*10^7$	$1.72887*10^7$	$0.94551*10^7$
V_{39}	$5.64247*10^7$	$41.6132*10^7$	$1.75899*10^7$	$1.57934*10^7$	$4.71445*10^7$
V_{40}	$6.01043*10^7$	$45.2504*10^7$	$1.81747*10^7$	$1.74186*10^7$	$1.22401*10^7$
V_{41}	$6.00204*10^7$	$45.0213*10^7$	$1.7939*10^7$	$1.719*10^7$	$1.37709*10^7$

We are observing that the N set is large enough; a final solution could be hard to find. We could choose the nondominated program situated closest distance to the ideal solution. However the founded solution could be unsatisfactory for the decision maker because it is not permitting a solid financial analysis and it not respects some qualitative criterions which couldn't be comprised in the model.

It's obvious that for determining the optimal solution we could apply the ELECTRE method, but the computations are too large in case of a great number of solutions.

Because the Nex set is to large I propose to limit it by imposing an acceptable threshold (an acceptable distance by the ideal solution) to that the variants are accepted by the decision maker. So became efficiently the use of ELECTRE method to determine the final solution, resulting $V^*= V_{38.}$ [$d_{38} = 0.94551*10^7$].

V_{38}:

Specification		Results	Observations
The values of the objectives functions	min. consumed energy kwh	$6.19533*10^7$	$+0.69175*10^7$
	max. produced energy, kwh	$45.7454*10^7$	$-0.0786*10^7$
	max. val. net prod. , *1000 lei	$1.79886*10^7$	$-0.09065*10^7$
	max. brut benefit , *1000 lei	$1.72887*10^7$	$-0.08451*10^7$
The structure of breeding and production technology	Wheat :X_1-X_6,ha	X_6=2159.07	< 3600
	Barley: X_7-X_{12},ha	X_{10}=2000	=2000
	Corn: X_{13}-X_{18} ,ha	X_{17}=2150.84 X_{18}=2730.09	<5200
Necessary resource	total budget, *1000 lei	34293521	-706479
	material budget, *1000 lei	28340100	-159900
	fertilizer .with nitrogen, t/ha.	1206.51	-293.49
	fertilizer .with phosphorus, t/ha.	577.54	-72.4613
	salary budget, *1000 lei	5953422	-546578
	manual man power day. hour	75000	0
	mechanized man power day. hour	15690.664	-309.336
	gas l	950000	0
The quantities which can be obtained	Wheat amount [t]	9253.76	+3153.76
	Barley amount [t]	9788	+7688
	Corn amount [t]	39403.8	+36903.8

Considering an acceptable threshold [$d=1.12*10^7$] results V_{38}, V_{22}, V_3, V_{36} şi V_{28}.
From the ELECTRE method the V_{28} is the final solution:

V_{28}

Specification		Results	Observations
The values of the objectives functions	min. consumed energy kwh	$6.1546*10^7$	$+0.69175*10^7$
	max. produced energy, kwh	$45.5296*10^7$	$-0.0786*10^7$
	max. val. net prod. , *1000 lei	$1.79527*10^7$	$-0.09065*10^7$
	max. brut benefit , *1000 lei	$1.74167*10^7$	$-0.08451*10^7$
The structure of breeding and production technology	Wheat :X_1-X_6,ha	X_5=2228.12	< 3600
	Barley: X_7-X_{12},ha	X_{10}=2000	=2000
	Corn: X_{13}-X_{18} ,ha	X_{17}=1858.73 X_{18}=2953.15	<5200
Necessary resource	total budget, *1000 lei	34017337	-982663
	material budget, *1000 lei	28034899	-465101
	fertilizer .with nitrogen, t/ha.	1198.362	-301.638
	fertilizer .with phosphorus, t/ha.	576.1577	-73.8423
	salary budget, *1000 lei	5982437	-517563
	manual man power day. hour	75000	0
	mechanized man power day. hour	15661.148	-338.852
	gas l	950000	0
The quantities which can be obtained	Wheat amount [t]	9549.7	+3449.7
	Barley amount [t]	9788	+7688
	Corn amount [t]	38846.3	+18346.3

It could be seen that it is difficult to choose the final solution. The great improvement of this method is the fact that permits to the decision maker a great freedom of movement between solutions which practically is all reasonable. Obvious the human decision which engage the future could not be inspired by only a strict mathematically prevision and in this meaning I'm conclude citing the remarkable Keynes paper ' Treaty about the money': 'we presume in the analysis perfect previsions but we know in fact that the prevision is imperfect'.

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