Acta Universitatis Apulensis No. 35/2013
ISSN: 1582-5329 pp. 125-131

DECOMPOSITION OF αM-CONTINUITY VIA IDEALS

S. Modak

Abstract. This paper will discuss about decomposition of α-M-continuity. For this, we have defined two new types of continuity on ideal minimal spaces and have obtained relationships with earlier continuities.

2000 *Mathematics Subject Classification*: 54A05, 54C10.

Keywords: M-continuous, αM-continuity, *M-precontinuity*, m-I-continuity, M-I-continuity

1. Introduction

The generalization of topology and its study are not a new concept in literature. Generalized Topology (GT) [1, 2, 3], is one of this generalization which has been introduced by Csaszar through function's approach. However Supratopology [7, 13] and Weak Structure [1] ware introduced from topology. Minimal Structure is also another generalization, this had been introduced by Maki et al [5, 6]. Further, the authors like Popa and Noiri [15, 16][15,16], Min and Kim [8, 9, 10, 11, 12] and Ozbakir et al [14] have studied it in detail.

In this paper we considered the minimal structure and the joint venture of ideal [4] and minimal structure on a nonempty set. Here we have characterized the αM-continuity with the help of ideals. For this, we define two types of set and continuities and discuss their relationships. Finally we have reached to the decomposition of αM-continuity.

2. Preliminaries

Definition 1. *[5, 6] A subfamily m_X of the power set $P(X)$ of a nonempty set X is called a minimal structure on X if $\emptyset \in m_X$ and $X \in m_X$. By (X, m_X), we denote a nonempty set X with a minimal structure m_X on X.*

Simply we call (X, m_X) a space with a minimal structure m_X on X. Set $M(x) = \{U \in m_X : x \in U\}$.

Theorem 1. *[5, 6] Let (X, m_X) be a space with a minimal structure m_X on X, for a subset A of X, the closure of A and the interior of A are defined as the following:*
(1) $mint(A) = \cup\{U : U \subseteq A, U \in m_X\}$.
(2) $mcl(A) = \cap\{F : A \subseteq F, X - F \in m_X\}$.

Theorem 2. *[5, 6] Let (X, m_X) be a space with a minimal structure m_X on X and $A \subseteq X$.*
(1) $X = mint(X)$ and $\emptyset = mcl(\emptyset)$.
(2) $mint(A) \subseteq A$ and $A \subseteq mcl(A)$.
(3) If $A \in m_X$, then $mint(A) = A$ and if $X - F \in m_X$, then $mcl(F) = F$.
(4) If $A \subseteq B$, then $mint(A) \subseteq mint(B)$ and $mcl(A) \subseteq mcl(B)$.
(5) $mint(mint(A)) = mint(A)$ and $mcl(mcl(A)) = mcl(A)$.
(6) $mcl(X - A) = X - mint(A)$ and $mint(X - A) = X - mcl(A)$.

Definition 2. *[15] Let (X, m_X) and (Y, m_Y) be two spaces with minimal structures m_X and m_Y, respectively. Then $f : X \to Y$ is said to be M-continuous if for $x \in X$ and $V \in M(f(x))$, there is $U \in M(x)$ such that $f(U) \subseteq V$.*

Definition 3. *[9] Let (X, m_X) be a minimal structure. A subset A of X is called an m-semiopen if $A \subseteq mcl(mint(A))$.*

The complement of an *m-semiopen* set is called an *m-semiclosed* set. The family of all *m-semiopen* sets in X will be denoted by $MSO(X)$.

Definition 4. *[9] Let $f : (X, m_X) \to (Y, m_Y)$ be a function between two spaces with minimal structures m_X and m_Y, respectively. Then f is said to be M-semicontinuous if for each x and each m-open set V containing $f(x)$, there exists an m-semiopen set U containing x such that $f(U) \subseteq V$.*

Theorem 3. *[9] Let $f : (X, m_X) \to (Y, m_Y)$ be a function on two spaces with minimal structures m_X and m_Y, respectively. Then f is M-semicontinuous if and only if $f^{-1}(V)$ is m-semiopen for each m-open set V in Y.*

Definition 5. *[8] Let (X, m_X) be a minimal structure. A subset A of X is called an αm-open set if $A \subseteq mint(mcl(mint(A)))$.*

The complement of an αm-open set is called an αm-closed set. The family of all αm-open sets in X will be denoted by $\alpha M(X)$.

Definition 6. *[8] Let $f : X \to Y$ be a function between minimal structures (X, m_X) and (Y, m_Y). Then f is said to be αM-continuous if for each x and each m-open set V containing $f(x)$, there exists an αm-open set U containing x such that $f(U) \subseteq V$.*

Theorem 4. *[8] Let $f : X \to Y$ be a function on two minimal structures (X, m_X) and (Y, m_Y). Then f is αM-continuous if and only if $f^{-1}(V)$ is an αm-open set for each m-open set V in Y.*

Definition 7. *[11] Let (X, m_X) be a minimal structure. A subset A of X is called an m-preopen set if $A \subseteq mint(mcl(A))$.*

A set A is called an *m-preclosed* set if the complement of A is *m-preopen* sets in X will be denoted by $MPO(X)$.

Definition 8. *[11] Let $f : X \to Y$ be a function between minimal structures (X, m_X) and (Y, m_Y). Then f is said to be M-precontinuous if for each x and each m-open set V containing $f(x)$, there exists an m-preopen set U containing x such that $f(U) \subseteq V$.*

Theorem 5. *[11] Let $f : X \to Y$ be a function on two minimal structures (X, m_X) and (Y, m_Y). Then f is M-precontinuous if and only if $f^{-1}(V)$ is an m-preopen set for each m-open set V in Y.*

Let I be an ideal [4] on X and m_X be a minimal structure on X, then (X, m_X, I) is called an ideal minimal space [14].

Definition 9. *[14] Let (X, m_X, I) be an ideal minimal space and $(.)_*$ be a set operator from $P(X)$ to $P(X)$. For a subset $A \subseteq X$, $A_*(I, m_X) = \{x \in X : U \cap A \notin I,$ for every $U \in M(x)\}$ is called minimal local function of A with respect to I and m_X. We will simply write A_* for $A_*(I, m_X)$.*

Definition 10. *[14] Let (X, m_X, I) be an ideal minimal space. Then the set operator m-cl* is called a minimal $*$-closure and is defined as m-cl$^*(A) = A \cup A_*$ for $A \subseteq X$. We will denoted by $m^*_X(I, m_X)$ the minimal structure generated by m-cl*, that is, $m^*_X(I, m_X) = \{U \subseteq X :$ m-cl$^*(X - U) = X - U\}$.*

$m^*_X(I, m_X)$ is called $*$-minimal structure which is finer than m_X. The elements of $m^*_X(I, m_X)$ are called minimal $*$-open(briefly, m^*-open) and the complement of an m^*-open set is called minimal $*$-closed(briefly, m^*-closed). Throughout the paper we simply m^*_X for $m^*_X(I, m_X)$.

Definition 11. *[14] A subset A of an ideal minimal space (X, m_X, I) is m^*-dense in itself(resp. m^*-perfect) if $A \subseteq A_*$(resp. $A_* = A$).*

Remark 1. *[14] A subset A of an ideal minimal space (X, m_X, I) is m^*-closed if and only if $A_* \subseteq A$.*

3. Continuity on ideal minimal spaces

Definition 12. *Let (X, m_X, I) be an ideal minimal space. A subset A of X is called an m-I-open set if $A \subseteq mint((A)_*)$.*

The family of all m-I-open sets in X will be denoted by $MIO(X)$.

Theorem 6. *Let (X, m_X, I) be an ideal minimal space. Any union of m-I-open sets is m-I-open.*

Proof. Let A_i be an m-I-open set for $i \in J$. Then $A_i \subseteq mint((A_i)_*) \subseteq mint((\cup A_i)_*)$. This implies $\cup_i A_i \subseteq mint((\cup A_i)_*)$. Hence $\cup_i A_i \in MIO(X)$.

It is obvious from above discussion, $MIO(X)$ forms a GT [1, 2, 3].

Theorem 7. *Let (X, m_X, I) be an ideal minimal space and $A \subseteq X$. If $A \in MIO(X)$ then $A \in MPO(X)$.*

Proof. It is obvious

Hence we have $MIO(X) \subseteq MPO(X)$, but reverse inclusion need not hold in general.

Remark 2. *Let $X = \{a, b, c, d\}$, $m_X = \{\emptyset, X, \{a\}, \{b\}, \{a, b, c\}, \{b, c\}, \{a, c\}\}$, $I = \{\emptyset, \{a\}\}$. For $A = \{a, c\}$, $A \subset mint(mcl(A))$, but $A_* = \{c, d\}$. Therefore $A \notin MIO(X)$.*

Theorem 8. *Let (X, m_X, I) be an ideal minimal space and $A \subseteq X$. If $A \in MIO(X)$, then A is m^*-dense in itself.*

Definition 13. *Let $f : X \to Y$ be a function between ideal minimal structures (X, m_X, I) and (Y, m_Y, J). Then f is said to be m-I-continuous if for each x and each m-open set V containing $f(x)$, there exists an m-I- open set U containing x such that $f(U) \subseteq V$.*

Theorem 9. *Let $f : X \to Y$ be a function between two ideal minimal spaces (X, m_X, I) and (Y, m_Y, J). Then f is m-I-continuous if and only if $f^{-1}(V)$ is an m-I- open set for each m-open set V in Y.*

Proof. Let f be m-I-continuous. Then for any m-open set V in Y and for each $x \in f^{-1}(V)$, there exists an m-I-open set U containing x such that $f(U) \subseteq V$. This implies $x \in U \subseteq f^{-1}(V)$ for each $x \in f^{-1}(V)$. Since any union of m-I-open sets is m-I-open, $f^{-1}(V)$ is m-I-open.
Converse part: Let $x \in X$ and for each m-open set V containing $f(x)$, $x \in f^{-1}(V) \subset mint((f^{-1}(V))_*)$. So there exists an m-I-open set U containing x such that $x \in U \subseteq f^{-1}(V)$, i.e., $f(U) \subseteq V$. Hence f is m-I-continuous.

Corollary 10. *Let $f : X \to Y$ be a function between two ideal minimal spaces (X, m_X, I) and (Y, m_Y, J). If f is m-I continuous then f is M-precontinuous.*

From Remark 2, the converse of this corollary need not hold in general.

Theorem 11. *Let $f : X \to Y$ be a m-I-continuous function between two ideal minimal spaces (X, m_X, I) and (Y, m_Y, J). Then $f^{-1}(V)$ is a m^*-dense in itself, for each m-open set V in Y.*

Proof. Proof is obvious from Theorem 8.

Definition 14. *Let (X, m_X, I) be an ideal minimal space. A subset A of X is called an M-I-open set if $A \subseteq (mint(A))_*$.*

The family of all M-I-open sets in X will be denoted by $MMIO(X)$.

Theorem 12. *Let (X, m_X, I) be an ideal minimal space. Any union of M-I-open sets is M-I-open.*

Proof. Let A_i be an M-I-open set for $i \in J$. Then $A_i \subseteq (mint(A_i))_* \subseteq (mint(\cup A_i))_*$. This implies $\cup_i A_i \subseteq (mint(\cup A_i))_*$. Hence $\cup_i A_i \in MMIO(X)$.

From above, it is obvious that $MMIO(X)$ forms a GT.

Theorem 13. *Let (X, m_X, I) be an ideal minimal space and $A \subseteq X$. If $A \in MMIO(X)$ then $A \in MSO(X)$.*

Proof. It is obvious.

Therefore we have $MMIO(X) \subseteq MSO(X)$. But following example shows that the converse inclusion need not hold in general.

Remark 3. *Let $X = \{a, b, c, d\}$, $m_X = \{\emptyset, X, \{a\}, \{b\}, \{a, b, c\}, \{b, c\}, \{a, c\}\}$, $I = \{\emptyset, \{a\}\}$. For $A = \{a, c\}, A \subset mcl(mint(A))$, but $(mint(A))_* = \{c, d\}$. Therefore $A \notin MMIO(X)$.*

Theorem 14. *Let (X, m_X, I) be an ideal minimal space and $A \subseteq X$. If $A \in MMIO(X)$, then A is m^*-dense in itself.*

Hence we have obtained following diagram:

$$m\text{-}I\text{-open} \Longrightarrow m^*\text{-dense in itself} \Longleftarrow M\text{-}I\text{-open}$$

Definition 15. *Let $f : X \to Y$ be a function between ideal minimal spaces (X, m_X, I) and (Y, m_Y, J). Then f is said to be M-I continuous if for each x and each m-open set V containing $f(x)$, there exists an M-I open set U containing x such that $f(U) \subseteq V$.*

Theorem 15. *Let $f : X \to Y$ be a function between two ideal minimal spaces (X, m_X, I) and (Y, m_Y, J). Then f is M-I continuous if and only if $f^{-1}(V)$ is an M-I open set for each m-open set V in Y.*

Proof. Let f be M-I-continuous. Then for any m-open set V in Y and for each $x \in f^{-1}(V)$, there exists an M-I-open set U containing x such that $f(U) \subseteq V$. This implies $x \in U \subseteq f^{-1}(V)$ for each $x \in f^{-1}(V)$. Since any union of M-I-open sets is M-I-open, $f^{-1}(V)$ is M-I-open.
Converse part: Let $x \in X$ and for each m-open set V containing $f(x)$, $x \in f^{-1}(V) \subset (mint(f^{-1}(V)))_*$. So there exists an M-I-open set U containing x such that $x \in U \subseteq f^{-1}(V)$, i.e., $f(U) \subseteq V$. Hence f is M-I-continuous.

Corollary 16. *Let $f : X \to Y$ be a function between two ideal minimal spaces (X, m_X, I) and (Y, m_Y, J). If f is M-I-continuous then f is M-semicontinuous.*

Proof. From Remark 3, the converse of this corollary need not hold in general.

Theorem 17. *Let $f : X \to Y$ be a M-I-continuous function between two ideal minimal spaces (X, m_X, I) and (Y, m_Y, J). If f is M-I continuous then $f^{-1}(V)$ is m^*-dense in itself, for each m-open set V in Y.*

Theorem 18. *Let $f : (X, m_X) \to (Y, m_Y)$ be a αM-continuous function. Then*
(1) f is M-semicontinuous; and
(2) f is M-precontinuous.

For reverse part of the this theorem, we get following:

Theorem 19. *Let $f : (X, m_X) \to (Y, m_Y)$ be a M-semicontinuous and M-precontinuous function. Then f is αM-continuous.*

Following corollary is a decomposition of αM-continuity.

Corollary 20. *Let $f : (X, m_X) \to (Y, m_Y)$ be a function. Then f is αM-continuous if and only if f is M-semicontinuous and M-precontinuous.*

Theorem 21. *Let $f : X \to Y$ be a function between ideal minimal spaces (X, m_X, I) and (Y, m_Y, J). If f is M-I-continuous and M-precontinuous, then f is αM-continuous.*

Reverse part of this theorem need not hold in general, because the concept of M-I-open sets and αm-open are different.

Theorem 22. *Let $f : X \to Y$ be a function between ideal minimal spaces (X, m_X, I) and (Y, m_Y, J). If f is M-I-continuous and m-I-continuous, then f is αM-continuous.*

Acknowledgements. The author is thankful to Prof. W. K. Min for his encouragement and cooperation.

References

[1] A. Csaszar, *Weak structures*, Acta Math. Hungar., 131, (1-2)(2011), 193 - 195.

[2] A. Csaszar, *Generalized topology, generalized continuity*, Acta Math. Hungar. 96, (4)(2002), 351 - 357.

[3] A. Csaszar, *Generalized open sets*, Acta Math. Hungar., 75, (1-2)(1997), 65 - 87.

[4] K. Kuratowski, *Topology*, Vol. I, Academic Press, New York, 1966.

[5] H. Maki, *On generalizing semi-open sets and preopen sets*, In: Meeting on Topological Spaces Theory and its Appication, August 1996, 13 -18.

[6] H. Maki, J. Umehara, and T. Noiri, *Every topological space is pre* $T_{1/2}$, Men. Fac. Sci. Kochi. Univ. Ser. A Math., 17, (1996), 33 - 42.

[7] A. S. Mashhour, A. A. Allam, F. S. Mahmoud, and Khadr, *On supratopological spaces*, Indian J. Pure Appl. Math. 14, (4)(1983), 502 - 510.

[8] W. K. Min, *αm-open sets and αM-continuous functions*, Commun. Korean Math. Soc. 25, 2(2010), 251 - 256.

[9] W. K. Min, *m-Semiopen sets and M-Semicontinous functions on spaces with minimal structures*, Honam Math. J., 31, 2(2009), 239 - 245.

[10] W. K. Min, *The generalized open sets on supratopology*, Kang-Kyun Math. Jour. 10, 1(2002), 25 -28.

[11] W. K. Min and Y. K. Kim, *m-preopen sets and M-precontinuity on spaces with minimal structures*, Advances in Fuzzy Sets and System, 4, 3(2009), 237 - 245.

[12] W. K. Min and Y. K. Kim, *On minimal precontinuous functions*, J. Chun Math. Soc., 22, 4(2009), 667 - 673.

[13] S. Modak and S. Mistry, *Ideal on supra topological space*, Int. J. Math. Analysis, 6, 1(2012), 1 - 10.

[14] O. B. Ozbakir and E. D. Yildirim, *On some closed sets in ideal minimal spaces*, Acta Math. Hungar., 125, 3(2009), 227 - 235.

[15] V. Popa and T. Noiri, *On M-continuous functions*, Anal. Univ. "Dunarea de Jos" Galati, Ser. Mat. Fiz. Mec. Teor. Fasc. II, 18, 23(2000), 31 - 41.

[16] V. Popa and T. Noiri, *On the definition of some generalized forms of continuity under minimal conditions*, Men. Fac. Scr. Kochi. Univ. Ser. Math. 22, (2001), 9 - 19.

Shyamapada Modak
Department of Mathematics, University of Gour Banga,
P.O. Mokdumpur, Malda - 732103
West Bengal, India
email: *spmodak2000@yahoo.co.in*