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SOME PROPERTIES OF CONVOLUTION FOR HYPERGEOMETRIC DISTRIBUTION TYPE SERIES ON CERTAIN ANALYTIC UNIVALENT FUNCTIONS

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Abstract. The purpose of the present paper is to obtain certain sufficient conditions for hypergeometric distribution type series on certain analytic univalent functions.

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1. Introduction

Let $\mathcal{A}$ denote the class of functions f of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1}$$

which are analytic in the open unit disc $U = \{z : z \in C \text{ and } |z| < 1\}$ and satisfy the normalization condition $f(0) = f'(0) - 1 = 0$. Further, we denote by S the subclass of $\mathcal{A}$ consisting of functions of the form (1) which are also univalent in U.

A function $f \in \mathcal{A}$ is said to be in the class $R^{\tau}(A, B)$, if

$$\left| \frac{f'(z) - 1}{\tau(A-B) - B(f'(z) - 1)} \right| < 1, \qquad (-1 \leq B < A \leq 1; \tau \in C/\{0\}; z \in U), \tag{2}$$

clearly, a function f belong to $R^{\tau}(A, B)$, if and only if there exist a function ω regular in U satisfying $\omega(0) = 0$ and

$$|\omega(z)| < 1, \qquad (z \in U) \text{ such that}$$

$$1+\frac{1}{\tau}(f'(z)-1)=\frac{1+A\omega(z)}{1+B\omega(z)}, \qquad (z\in U), \tag{3}$$

the class $R^{\tau}(A,B)$ was introduced by Dixit and Pal [3].

A function f of the form (1) is said to be starlike of order α if it satisfies the following condition

$$\Re\left\{\frac{zf'(z)}{f(z)}\right\}>\alpha, \qquad z\in U,$$

the classes of all starlike functions are denoted by $S^*(\alpha)$, studied by Robertson [14] and Silverman [15].

For $\lambda>0$, Ponnusamy and Rønning [10] introduced the classes S^*_λ and C_λ consisting of functions of the form (1) as follows

$$S^*_\lambda=\left\{f\in\mathcal{A}:\left|\frac{zf'(z)}{f(z)}-1\right|<\lambda, \quad z\in U\right\},$$

and

$$C_\lambda=\left\{f\in\mathcal{A}:zf'(z)\in S^*_\lambda\right\}.$$

It is obvious that $f\in C_\lambda$ if and only if $zf'(z)\in S^*_\lambda$.

Recently, Porwal [11] gives a beautiful application of Poission distribution series on these subclasses. They establishes a connection between probability distribution and complex analysis and opened up a new direction of research in Geometric Function Theory. After the appearence of this paper many researchers e.g. (Murugusundaramoorthy [8], Murugusundaramoorthy *et al.* [9], Porwal and Kumar [12], [13] etc.) obtained certain necessary and sufficient conditions for Poisson distribution series, confluent hypergeometric distribution series, hypergeometric distribution type series etc. belonging to the various subclasses of univalent functions. Motivating with the above mentioned work, we obtain neccessary and sufficient conditions for hypergeometric distribution type series belonging to the classes $R^{\tau}(A,B)$, S^*_λ, C_λ, UCV, UST and $k-UCV(\alpha)$.

Very recently, Ahmad [1] introduce hypergeometric type distribution as follows, for this purpose we recall the definition of hypergeometric series. The power series

$$\sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n(1)_n}z^n,$$

where a, b, c are complex numbers such that $c \neq 0$, -1, -2,...... and $(a)_n$ is the Pochhammer symbol defined in terms of the Gamma function by,

$$\begin{aligned}(a)_n &= \frac{\Gamma(a+n)}{\Gamma(a)} \\ &= \begin{cases} 1, & \text{if } n = 0 \\ a(a+1)\dots(a+n-1), & \text{if } n \in N = \{1,2,3,\dots\} \end{cases}\end{aligned}$$

is convergent for all finite value of z. is called the Hypergeometric series. The series converges absolutly if $|z| < 1$ and diverges if $|z| > 1$ and for $|z| = 1$ the series is absolutly convergent if $\Re(c - a - b) > 0$. It is denoted by $F(a, b; c; z)$.

Now we define for $a, b, c, m > 0$ such that the series,

$$F(a, b; c; m) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n (1)_n} m^n,$$

is convergent.

Now we introduce hypergeometric type distribution whose probability mass function is,

$$\frac{(a)_n (b)_n}{(c)_n n! F(a, b; c; m)} m^n.$$

Now we introduce a new series $I(a, b; c; m; z)$ in the following way,

$$I(a, b; c; m; z) = z + \sum_{n=2}^{\infty} \frac{(a)_{n-1} (b)_{n-1} m^{n-1}}{(c)_{n-1} (n-1)! F(a, b; c; m)} z^n,$$

where $a, b, c, m > 0$. The series $I(a, b; c; m; z)$ is absolutly convergent for $|z| < 1$.

The convolution (or Hadamard product) of two series

$$f(z) = \sum_{n=0}^{\infty} a_n z^n,$$

and

$$g(z) = \sum_{n=0}^{\infty} b_n z^n,$$

is defined as the power series,

$$(f * g)(z) = \sum_{n=0}^{\infty} a_n b_n z^n.$$

Now we consider a linear operator,

$$K(a,b;c;m;z)f(z) = z + \sum_{n=2}^{\infty} \frac{(a)_{n-1}(b)_{n-1}m^{n-1}}{(c)_{n-1}(n-1)!F(a,b;c;m)} a_n z^n.$$

Bharti *et al.* [2] introduced the subclass of k-uniformly convex functions of order α and corressponding class of starlike functions as follows-

If $f \in \mathcal{A}$, $0 \leq k < \infty$ and $0 \leq \alpha < 1$ then $f \in k-UCV(\alpha)$, if and only if

$$\Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} \geq k\left|\frac{zf''(z)}{f'(z)}\right| + \alpha. \tag{4}$$

For $\alpha = 0$ the class $k-UCV(\alpha)$ reduce to the class $k-UCV$ introduced and studied by Kanas and Wisniowska [6] and for $k = 1, \alpha = 0$ it reduce to the class uniformly convex function UCV studied by Goodman [5], (see also [4], [7] and [16]).
To prove our main results we shall require the following lemmas.

Lemma 1. *([3]) Let a function f of the form (1) be in $R^{\tau}(A,B)$. Then,*

$$|a_n| \leq \frac{(A-B)|\tau|}{n}.$$

The result is sharp for the function

$$f(z) = \int_0^z \left(1 + \frac{(A-B)\tau z^{n-1}}{1+Bz^{n-1}}\right) dz, \qquad (n \geq 2; z \in U). \tag{5}$$

Lemma 2. *([3]) Let a function f of the form (1) be in $\mathcal{A}$. If*

$$\sum_{n=2}^{\infty} (1+|B|)n|a_n| \leq (A-B)|\tau|, \qquad (-1 \leq B < A \leq 1; \tau \in C).$$

Then $f \in R^{\tau}(A,B)$.

The result is sharp for the function

$$f(z) = z + \frac{(A-B)\tau}{(1+|B|)n} z^n, \qquad (n \geq 2; z \in U). \tag{6}$$

Lemma 3. *([10]) Let $f \in A$ be of the form (1). If*

$$\sum_{n=2}^{\infty} (\lambda + n - 1)|a_n| \leq \lambda, \quad (\lambda > 0), \tag{7}$$

then $f \in S^*_\lambda$.

Lemma 4. *[10] Let $f \in A$ be of the form (1). If*

$$\sum_{n=2}^{\infty} n(\lambda+n-1)|a_n| \leq \lambda, \quad (\lambda>0), \tag{8}$$

then $f \in C_\lambda$.

Lemma 5. *([5]) A function f of the form (1) is in UCV if*

$$\sum_{n=2}^{\infty} n(2n-1)|a_n| \leq 1.$$

Lemma 6. *([5]) A function f of the form (1) is in UST if*

$$\sum_{n=2}^{\infty} (3n-2)|a_n| \leq 1.$$

Lemma 7. *([2]) A function $f \in A$ is in $k-UCV(\alpha)$ if it satisfies the following condition*

$$\sum_{n=2}^{\infty} n[n(1+k)-(k+\alpha)]|a_n| \leq 1-\alpha. \tag{9}$$

2. Main Results

Theorem 8. *Let $a, b, c > 0$ and $c > a+b$, $m \in (0,1)$. Suppose that $f \in R^\tau(A,B)$ and satisfy the condition*

$$F(a,b;c;m) \leq 1 + \frac{1}{|B|},$$

then the operator $K(a,b;c;m;z)$ maps $R^\tau(A,B)$ into $R^\tau(A,B)$.

Proof. Let $a, b, c > 0$ and $m \in (0,1)$.
Suppose that $f(z) = z + \sum_{n=2}^{\infty} a_n z^n \in R^\tau(A,B)$.
Then by Lemma 2, it sufficeint to show that,

$$T_1 = \sum_{n=2}^{\infty} (1+|B|)n|A_n| \leq (A-B)|\tau|,$$

where
$$A_n=\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}\frac{m^{n-1}}{F(a,b;c;m)}a_n.$$

$$\begin{aligned}T_1&=\sum_{n=2}^{\infty}(1+|B|)n\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}\frac{m^{n-1}}{F(a,b;c;m)}|a_n|\\
&\le(1+|B|)\sum_{n=2}^{\infty}\frac{n(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}\frac{m^{n-1}}{F(a,b;c;m)}\frac{(A-B)|\tau|}{n}\\
&=\frac{(A-B)|\tau|(1+|B|)}{F(a,b;c;m)}\sum_{n=2}^{\infty}\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}m^{n-1}\\
&=\frac{(A-B)|\tau|(1+|B|)}{F(a,b;c;m)}(F(a,b;c;m)-1)\\
&\le(A-B)|\tau|,\end{aligned}$$

by the given hypothesis.

Thus the operator $K(a,b;c;m;z)$ maps $R^{\tau}(A,B)$ into $R^{\tau}(A,B)$.

Theorem 9. *Let $a,b,c>1$ and $c>a+b$, $m\in(0,1)$. Suppose that $f\in R^{\tau}(A,B)$ and satisfy the condition*

$$\frac{|A-B||\tau|}{F(a,b;c;m)}\{F(a,b;c;m)-1\}+\frac{(\lambda-1)(c-1)}{(a-1)(b-1)m}\left\{F(a-1,b-1;c-1;m)-1-\frac{(a-1)(b-1)}{(c-1)}m\right\}\le\lambda,$$

then the operator $K(a,b;c;m;z)$ maps $R^{\tau}(A,B)$ into S^*_{λ}.

Proof. Let $a,b,c>1$ and $m\in(0,1)$. Suppose that $f(z)=z+\sum_{n=2}^{\infty}a_nz^n\in R^{\tau}(A,B)$. Then by Lemma 3 it is sufficient to show that

$$\begin{aligned}T_2&=\sum_{n=2}^{\infty}(n+\lambda-1)|A_n|\le\lambda\\
&=\sum_{n=2}^{\infty}(n+\lambda-1)\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}\frac{m^{n-1}}{F(a,b;c;m)}|A_n|\\
&=\sum_{n=2}^{\infty}(n+\lambda-1)\left|\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}m^{n-1}\right|\frac{|A-B||\tau|}{nF(a,b;c;m)}\\
&=\frac{(A-B)|\tau|}{F(a,b;c;m)}\left\{\sum_{n=2}^{\infty}\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}m^{n-1}+(\lambda-1)\sum_{n=2}^{\infty}\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n)!}m^{n-1}\right\}\end{aligned}$$

$$= \frac{|A-B||\tau|}{F(a,b;c;m)}\left\{\sum_{n=1}^{\infty}\frac{(a)_n(b)_n}{(c)_n(n)!}m^n+(\lambda-1)\sum_{n=2}^{\infty}\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n)!}m^{n-1}\right\}$$

$$= \frac{|A-B||\tau|}{F(a,b;c;m)}\left\{\sum_{n=1}^{\infty}\frac{(a)_n(b)_n}{(c)_n(n)!}m^n+\frac{(\lambda-1)(c-1)}{(a-1)(b-1)m}\sum_{n=2}^{\infty}\frac{(a-1)_n(b-1)_n}{(c-1)_n(n)!}m^n\right\}$$

$$= \frac{|A-B||\tau|}{F(a,b;c;m)}\left\{F(a,b;c;m)-1\right\}+\frac{(\lambda-1)(c-1)}{(a-1)(b-1)m}$$

$$\left\{F(a-1,b-1;c-1;m)-1-\frac{(a-1)(b-1)}{(c-1)}m\right\}$$

$$\leq\lambda,$$

by the given hypothesis.
Thus the operator $K(a,b;c;m;z)$ maps $R^\tau(A,B)$ into S^*_λ.

Theorem 10. *Let $a,b,c>0$ and $c>a+b$, $m\in(0,1)$. Suppose that $f\in R^\tau(A,B)$ and satisfy the condition*

$$(A-B)|\tau|\left\{\frac{ab}{c}mF(a+1,b+1;c+1;m)+\lambda\left\{F(a,b;c;m)-1\right\}\right\}\leq\lambda F(a,b;c;m),$$

then the operator $K(a,b;c;m;z)$ maps $R^\tau(A,B)$ into C_λ.

Proof. Let $a,b,c>0$ and $m\in(0,1)$. Suppose that $f(z)=z+\sum_{n=2}^{\infty}a_nz^n\in R^\tau(A,B)$. Then by Lemma 4. It is sufficient to show that

$$T_3=\sum_{n=2}^{\infty}n(n+\lambda-1)|A_n|\leq\lambda.$$

Now

$$T_3=\sum_{n=2}^{\infty}n(n+\lambda-1)\left|\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}m^{n-1}\frac{a_n}{F(a,b;c;m)}\right|$$

$$\leq\sum_{n=2}^{\infty}n(n+\lambda-1)\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}\frac{m^{n-1}}{F(a,b;c;m)}\frac{(A-B)|\tau|}{n}$$

$$=\frac{(A-B)|\tau|}{F(a,b;c;m)}\left\{\sum_{n=2}^{\infty}\left\{(n-1)+\lambda\right\}\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}m^{(n-1)}\right\}$$

$$=\frac{(A-B)|\tau|}{F(a,b;c;m)}\left\{\sum_{n=2}^{\infty}(n-1)\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}m^{n-1}+\lambda\sum_{n=2}^{\infty}\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}m^{n-1}\right\}$$

$$= \frac{(A-B)|\tau|}{F(a,b;c;m)} \left\{ \sum_{n=2}^{\infty} \frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-2)!} m^{n-1} + \lambda \sum_{n=1}^{\infty} \frac{(a)_n(b)_n}{(c)_n(n)!} m^n \right\}$$

$$= \frac{(A-B)|\tau|}{F(a,b;c;m)} \left\{ \frac{ab}{c} m \sum_{n=2}^{\infty} \frac{(a+1)_{n-2}(b+1)_{n-2}}{(c+1)_{n-2}(n-2)!} m^{n-2} + \lambda \left\{ F(a,b;c;m) - 1 \right\} \right\}$$

$$= \frac{(A-B)|\tau|}{F(a,b;c;m)} \left\{ \frac{ab}{c} m F(a+1,b+1;c+1;m) + \lambda \left\{ F(a,b;c;m) - 1 \right\} \right\}$$

$$\leq \lambda,$$

by the given hypothesis.
Thus the operator $K(a,b;c;m;z)$ maps $R^{\tau}(A,B)$ into C_{λ}.

Theorem 11. *Let $a,b,c>0$ and $c>a+b$, $m \in (0,1)$. Suppose that $f \in R^{\tau}(A,B)$ and satisfy the condition*

$$(A-B)|\tau| \left\{ 2\frac{ab}{c} m F(a+1,b+1;c+1;m) + F(a,b;c;m) - 1 \right\} \leq F(a,b;c;m),$$

then the operator $K(a,b;c;m;z)$ maps $R^{\tau}(A,B)$ into UCV.

Proof. Let $a,b,c>0$ and $m \in (0,1)$. Suppose that $f(z) = z + \sum_{n=2}^{\infty} a_n z^n \in R^{\tau}(A,B)$. Then by Lemma 5, we have

$$T_4 = \sum_{n=2}^{\infty} n(2n-1) \frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!} \frac{m^{n-1}}{F(a,b;c;m)} |a_n|$$

$$\leq \sum_{n=2}^{\infty} n(2n-1) \frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!} m^{n-1} \frac{(A-B)|\tau|}{nF(a,b;c;m)}$$

$$= \frac{(A-B)|\tau|}{F(a,b;c;m)} \sum_{n=2}^{\infty} \left\{ 2(n-1)+1 \right\} \frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!} m^{n-1}$$

$$= \frac{(A-B)|\tau|}{F(a,b;c;m)} \left\{ 2\sum_{n=2}^{\infty} (n-1) \frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!} m^{n-1} + \sum_{n=2}^{\infty} \frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!} m^{n-1} \right\}$$

$$= \frac{(A-B)|\tau|}{F(a,b;c;m)} \left\{ 2\sum_{n=2}^{\infty} \frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-2)} m^{n-1} + \sum_{n=1}^{\infty} \frac{(a)_n(b)_n}{(c)_n(n)!} m^n \right\}$$

$$= \frac{(A-B)|\tau|}{F(a,b;c;m)} \left\{ 2\frac{ab}{c} m \sum_{n=0}^{\infty} \frac{(a+1)_n(b+1)_n}{(c+1)_n(n)!} m^n + F(a,b;c;m) - 1 \right\}$$

$$= \frac{(A-B)|\tau|}{F(a,b;c;m)}\left\{2\frac{ab}{c}mF(a+1,b+1;c+1;m)+F(a,b;c;m)-1\right\}$$
$$\leq 1,$$

by the given hypothesis.
Thus the operator $K(a,b;c;m;z)$ maps $R^{\tau}(A,B)$ into UCV.

Theorem 12. *Let $a,b,c>1$ and $c>a+b$, $m\in(0,1)$. Suppose that $f\in R^{\tau}(A,B)$ and satisfy the condition ,*

$$(A-B)|\tau|\left\{3F(a,b;c;m)-1+\frac{(c-1)m}{(a-1)(b-1)}2F(a-1,b-1;c-1;m)-1-\frac{(a-1)(b-1)}{(c-1)}\right\}$$
$$\leq F(a,b;c;m),$$

then the operator $K(a,b;c;m;z)$ maps $R^{\tau}(A,B)$ into UST.

Proof. Let $a,b,c>0$ and $m\in(0,1)$.
Suppose that $f(z)=z+\sum_{n=2}^{\infty}a_nz^n\in R^{\tau}(A,B)$.
Then by Lemma 6, it is sufficient to show that

$$T_5=\sum_{n=2}^{\infty}(3n-2)\left|\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}\frac{m^{n-1}}{F(a,b;c;m)}|a_n|\right|$$
$$=\sum_{n=2}^{\infty}(3n-2)\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}\frac{m^{n-1}}{F(a,b;c;m)}|a_n|$$
$$\leq\sum_{n=2}^{\infty}(3n-2)\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}\frac{m^{n-1}}{F(a,b;c;m)}\frac{|A-B||\tau|}{n}$$
$$=\frac{|A-B||\tau|}{F(a,b;c;m)}\left\{\sum_{n=2}^{\infty}3n\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n)!}m^{n-1}-2\sum_{n=2}^{\infty}\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n)!}m^{n-1}\right\}$$
$$=\frac{|A-B||\tau|}{F(a,b;c;m)}\left\{\sum_{n=2}^{\infty}3\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}m^{n-1}-2\sum_{n=2}^{\infty}\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}n!}m^{n-1}\right\}$$
$$=\frac{(A-B)|\tau|}{F(a,b;c;m)}\left\{3\sum_{n=1}^{\infty}\frac{(a)_n(b)_n}{(c)_nn!}m^n-2\frac{(c-1)}{(a-1)(b-1)}m\sum_{n=2}^{\infty}\frac{(a-1)_n(b-1)_n}{(c-1)_(n)n!}m^n\right\}$$
$$=\frac{(A-B)|\tau|}{F(a,b;c;m)}\left\{3(F(a,b;c;m)-1)+\frac{(c-1)}{(a-1)(b-1)m}2\right.$$
$$\left.\left(F(a-1,b-1;c-1;m)-1-\frac{(a-1)(b-1)}{(c-1)}m\right)\right\}$$

$$\leq 1,$$

by the given hypothesis.
Thus the operator $K(a,b;c;m;z)$ maps $R^{\tau}(A,B)$ into UST.

Theorem 13. *Let $a,b,c>0$ and $c>a+b$, $m\in(0,1)$. Suppose a function $f\in R^{\tau}(A,B)$ is in $k-UCV(\alpha)$ if it satisfies the following condition*

$$\frac{(A-B)|\tau|}{F(a,b;c;m)}[(k+1)\frac{ab}{c}mF(a+1,b+1;c+1;m)+(1-\alpha)[F(a,b;c;m)-1]]\leq(1-\alpha).$$

Then the operator $K(a,b;c;m;z)$ maps $k-UCV(\alpha)$.

Proof. Let $a,b,c>0$ and $c>a+b$. Suppose a function $f\in R^{\tau}(A,B)$ is in $k-UCV(\alpha)$,then by Lemma 7, it is sufficient to show that

$$T_6=\sum_{n=2}^{\infty}n[n(1+k)-(k+\alpha)]|A_n|\leq 1-\alpha$$

Now

$$T_6=\sum_{n=2}^{\infty}n[n(1+k)-(k+\alpha)]\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}\frac{m^{n-1}}{F(a,b;c;m)}(|a_n|)$$

$$\leq\frac{(A-B)|\tau|}{F(a,b;c;m)}\sum_{n=2}^{\infty}n[n(k+1)-(k+\alpha)]\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}\frac{m^{n-1}}{n}$$

$$=\frac{(A-B)|\tau|}{F(a,b;c;m)}\sum_{n=2}^{\infty}[(k+1)(n-1)+(1-\alpha)]\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}m^{n-1}]$$

$$=\frac{(A-B)|\tau|}{F(a,b;c;m)}[(k+1)\sum_{n=2}^{\infty}\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-2)!}m^{n-1}+(1-\alpha)\sum_{n=2}^{\infty}\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}m^{n-1}]$$

$$=\frac{(A-B)|\tau|}{F(a,b;c;m)}[(k+1)\frac{ab}{c}mF(a+1,b+1;c+1;m)+(1-\alpha)[F(a,b;c;m)-1]]$$

$$\leq(1-\alpha),$$

by the given hypothesis.
Thus the operator $K(a,b;c;m;z)$ maps $k-UCV(\alpha)$.

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