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Constancy of Maps into f-manifolds and pseudo f-manifolds

Sadettin Erdem

Matematik Bölümü, Anadolu Üniversitesi
Yunus Emre Yerleşkesi, Eskişehir, Türkiye
e-mail: s2erdem@anadolu.edu.tr

Abstract. Constancy of some maps into certain f-manifolds and pseudo f-manifolds is discussed. As a result a Constancy Theorem is given, for maps of Riemannian manifolds into these manifolds, which recovers all the results obtained so far.

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Introduction

In dealing with maps, such as harmonic and holomorphic ones, between manifolds existence question becomes an essential part of their study. Mathematicians normally study such maps under certain conditions imposed on the manifolds and on the maps themselves. A vital question is that whether there exist such maps under the restrictions imposed. We highlight a few result on this line:

i) If a harmonic map is constant on the boundary of a flat ball of any dimension then it is constant on the whole ball, [12].

ii) Let $\phi : M \to N$ be a holomorphic map between Kaehler manifolds with M compact and $rank(\phi) < \dim(M)$. If, for the respective Kaehler forms ω^M and ω^N, the cohomology classes satisfy that $[\phi^*\omega^N] = c[\omega^M]$ for some $c \in \mathbb{R}$ then ϕ is constant. In particular, one may take M to be a complex Grassmannian or a complex quadric, [6].

Almost a decade ago mathematicians (see for example [5], [7], [8], [9] and [11]) have started considering holomorphic maps and harmonic maps between (hyperbolic) metric (para) f-manifolds. The main difference of a (pseudo) f-manifold from the complex one is that an almost complex structure J on a manifold M is replaced with a more general one, an f-structure (resp. a pseudo f-structure), which is a 1-1 tensor field f satisfying $f^3+f=0$ (resp. $f^3-f=0$) with $rank(f) \leq \dim(M)$ whereas $rank(J)=\dim(M)$. In 2001, the following results are established:

iii) Every holomorphic map from an almost Hermitian manifold into an almost $\mathcal{S}$-manifold is constant, [5].

iv) Every holomorphic map from a semi-Kaehler manifold into a strongly pseudoconvex CR-manifold is constant, [11].

In this work we give a non-existence result, Theorem 2.2, for maps into certain pseudo f-manifolds as well as f-manifolds under more relaxed conditions. Our result, in turn, covers and improves the above last two results in [5], [11] and also in the author's earlier work [10]. The main features of our work are the ones that all restrictions on the domain manifold are removed and wide ranges of pseudo f-manifolds as well as f-manifolds in the target are covered. Also hypotheses of the main theorem in [10] are relaxed and simplified.

1. Preliminaries

For a smooth manifold $N^{k+\ell}$ let φ denote a $(1,1)$-tensor field on N of rank k and nullity $\ell \geq 0$. Put $D=\varphi(TN)$ and $\mathcal{V}=Ker(\varphi)$. The distributions D and $\mathcal{V}$ over N are called *φ-horizontal* and *φ-vertical of rank k and ℓ* respectively. The pair (N,φ) is then called an *f-manifold* (resp. *pseudo f-manifold* [13],[15]) provided

$$\varphi^3+\varphi=0 \quad (\text{resp. } \varphi^3-\varphi=0 \text{ with } \varphi\mid_D \neq I, \text{ the identity}).$$

An f-manifold (resp. a pseudo f-manifold) (N,φ) is called a *metric f-manifold* (resp. a *metric pseudo f-manifold*) if it carries Riemannian or semi-Riemannian metric h such that

i) $h(X,Y)=0\ \forall X\in D,\ Y\in\mathcal{V}$ that is, D and $\mathcal{V}$ are h-orthogonal;

ii) $h(X,Y)=h(\varphi X,\varphi Y)\ \forall\ X,\ Y\in D$.

We refer the conditions (i) and (ii) as *h-compatibility of φ*. Note here that the condition (i) implies that h is nondegenerate on $\mathcal{V}$ and therefore on D. In the case of a pseudo f-manifold (N,φ) if the condition (ii) reads instead,

ii)' $h(X,Y)=-h(\varphi X,\varphi Y)\ \forall\ X,Y\in D$

then (N,h,φ) is called a *hyperbolic metric pseudo f-manifold*. In this case, we refer the conditions (i) and (ii)' as *h-hyperbolic compatibility of φ*.

Note that the pseudo f-structure φ induces a decomposition on the tangent bundle:

$$TN=\mathcal{V}\oplus D^+\oplus D^-$$

into eigenbundles $\mathcal{V},\ D^+,\ D^-$ corresponding to eigenvalues 0, 1, and -1 respectively so that $\varphi(X)=X,\ \forall\ X\in D^+$ and $\varphi(X)=-X,\ \forall\ X\in D^-$. A pseudo f-manifold is also called a *para f-manifold* if $rank(D^+)=rank(D^-)$.

It is easily seen that

i) a hyperbolic metric pseudo f-manifold is necessarily a hyperbolic metric para f-manifold,

ii) the horizontal distribution D of an f-manifold has to be of even rank. But for a pseudo f-manifold, $rank(D)$ can be even or odd. However, as in the case of f-manifold, for a hyperbolic metric pseudo f-manifold, $rank(D)$ has to be even.

Throughout, the metrics considered on pseudo f-manifolds would always be hyperbolic ones so that the manifold (N, h, φ) would be either hyperbolic metric pseudo f-manifold (which is necessarily a hyperbolic metric para f-manifold) or a metric f-manifold. (See [4], for pseudo f-manifolds with non-hyperbolic metrics.) Thus, in this work, we would write metric para f-manifold to mean a hyperbolic metric para f-manifold.

Suppose there is a global frame field $\{\xi_j\}_{j=1}^{\ell}$ for the φ-vertical bundle $\mathcal{V}$ with their dual 1-forms $\{\eta^j\}_{j=1}^{\ell}$ satisfying

a°) $\quad \varphi^2 = r(-I + \sum_{j=1}^{\ell} \eta^j \otimes \xi_j), \quad \eta^j(\xi_a) = \delta_a^j$

b°) $\quad h(\varphi X, \varphi Y) = r\{h(X,Y) - \sum_{j=1}^{\ell} \eta^j(X)\eta^j(Y)\}$

where

$$r = \begin{cases} 1, & \text{if } \varphi \text{ is an } f\text{-structure,} \\ -1, & \text{if } \varphi \text{ is a pseudo } f\text{-structure,} \end{cases}$$

then $N = (N^{k+\ell}; h, \varphi, \xi_j, \eta^j)$ is called a *globally framed metric (para) f-manifold.* For the class of globally framed metric (para) f-manifolds $N = (N^{2n+\ell}; h, \varphi, \xi_j, \eta^j)$ we list some of its subclasses for later use ([1], [2], [3], [4], [5]):

i) For $\ell = 0$, $N = (N^{2n}, h, \varphi)$ is called an *almost (para) Hermitian manifold.*

ii) For $\ell \geq 1$, setting $\Omega(X,Y) = h(X, \varphi Y)$,

a°) N is called an *almost (para) $\mathcal{S}$-manifold* provided $d\eta^j = \Omega$, for each $j \in \{1, 2, \ldots, \ell\}$.

b°) N is called an *almost (para) $\mathcal{C}$-manifold* provided $d\Omega = 0$ and $d\eta^j = 0$ for each $j = 1, 2, \ldots, \ell$.

iii) If $\ell = 1$, then $N = (N^{2n+1}; h, \varphi, \xi, \eta)$ is called an *almost (para)contact metric manifold* (almost $(P)CM$-manifold). If further an almost $(P)CM$-manifold is also an almost (para) $\mathcal{S}$-manifold (so that $d\eta = \Omega$) then we drop the adjective 'almost' and simply call it a *(para)contact metric manifold.* If an almost $(P)CM$-manifold is also an almost (para) $\mathcal{C}$-manifold, that is $d\Omega = 0$ and $d\eta = 0$, then it is called *almost (para)cosymplectic.*

iv) An almost $(P)CM$-manifold $(N^{2n+1}, h, \varphi, \xi, \eta)$ is called

a°) *nearly (para) Sasakian* if $\forall X, Y \in \Gamma(TN)$

$$(\nabla_X \varphi)Y + (\nabla_Y \varphi)X = 2h(X,Y)\xi - r(\eta(X)Y + \eta(Y)X)$$

where ∇ denotes the Levi-Civita connection.

b°) *almost trans-Sasakian of type* (α, β) if

$$d\eta = \alpha\Omega - \frac{1}{n}\eta \wedge \varphi^*(\delta\Omega) \;\text{ and }\; d\Omega = \Omega \wedge (\frac{1}{n}\varphi^*(\delta\Omega) - \beta\eta)$$

for some functions α, β on N, where δ is the codifferential operator and $\varphi^*(\delta\Omega)(X) = (\delta\Omega)(\varphi X)$. In particular, if $\alpha = \frac{1}{2n}(\delta\Omega)(\xi)$ and $\beta = \frac{1}{n}\delta\eta = div(\xi)$ then N is simply called an *almost trans-Sasakian manifold*, [3], [15].

c°) *nearly (para) cosymplectic* if $(\nabla_X\varphi)X = 0,\ \forall X \in \Gamma(TN)$.

d°) *quasi-K-(para) cosymplectic* if

$$\mathcal{S}(X,Y) := (\nabla_X\varphi)Y + r(\nabla_{(\varphi X)}\varphi)(\varphi Y) = \eta(Y)\nabla_{(\varphi X)}\xi.$$

e°) *(para) cosymplectic* if it is almost (para) cosymplectic and normal (i.e. $N^1 = [\varphi,\, \varphi](X,Y) + r2d\eta(X,Y)\xi = 0$).

Remark. i) Every almost cosymplectic manifold is quasi K-cosymplectic, [14].
ii) Every almost cosymplectic manifold $(N^{2n+1}, h, \varphi, \xi, \eta)$ is cosymplectic, if and only if $\nabla\varphi = 0$, that is φ is parallel, [1].
iii) The above two statements are also valid for the "para" cases.

Let $\phi : (M,g) \to (N,h)$ be a smooth map between (semi) Riemannian manifolds. Set $\mathcal{K} = \mathcal{K}_\phi = ker(d\phi)$, $\mathcal{H} = \mathcal{H}_\phi = \mathcal{K}^\perp$ and $\overline{M} = \{p \in M : d\phi_p \neq 0\}$. If $\mathcal{K}$ (and therefore $\mathcal{H}$ forms a bundle then $\mathcal{K}$ and $\mathcal{H}$ are called *vertical* and *horizontal distributions associated with* ϕ respectively.

Throughout our work the map ϕ will be smooth and $\overline{M}$, when non-empty, will be a dense open subset of M

Definition 1.1. *A smooth map $\phi : M \to (N, \varphi)$ into an f-manifold [resp. a pseudo f-manifold] is said to be φ-invariant if $\varphi \circ d\phi(TM) = d\phi(TM)$ [resp. $\varphi \circ d\phi(TM) = d\phi(TM)$ and $d\phi(T_pM)$ is contained in neither $D^+(q)$ nor $D^-(q)$ $\forall\, p \in M$, where $q = d\phi(p)$]*

Remark. Note that
i) The φ-invariance of ϕ implies that $d\phi(TM) \subseteq {D^N}_\varphi$.
ii) Every φ-invariant map into a surface is necessarily a submersion of $\overline{M}$.

A smooth map $\phi : (M, \varphi^M) \to (N, \varphi^N)$ between (pseudo) f-manifolds is said to be (φ^M, φ^N)-*holomorphic* [resp. (φ^M, φ^N)-*antiholomorphic*] if

$$d\phi \circ \varphi^M = \varphi^N \circ d\phi, \quad \text{[resp. } d\phi \circ \varphi^M = -\varphi^N \circ d\phi].$$

We write $\pm(\varphi^M, \varphi^N)$-*holomorphic* to mean either (φ^M, φ^N)-holomorphic or (φ^M, φ^N)-antiholomorphic.

For a φ-invariant map $\phi : (M,g) \to (N, \varphi)$ of a Riemannian or semi Riemannian manifold into a (pseudo) f-manifold (not necessarily of constant rank) (with g nondegenerate on $\mathcal{K}$), set, $\forall\, p \in \overline{M}$,

$$\Psi_p(X_p) = \Psi_p^\phi(X_p) = \begin{cases} (d\phi)^{-1} \circ \varphi \circ d\phi(X_p) & , \quad X_p \in \mathcal{H}_p \\ 0 & , \quad X_p \in \mathcal{K}_p \end{cases} \tag{1.1}$$

where $d\phi^{-1} = (d\phi|_{\mathcal{H}})^{-1}$.

Observe that for every φ-invariant map $\phi : (M, g) \to (N, \varphi)$ of a Riemannian or semi Riemannian manifold into a (pseudo) f-manifold we have $d\phi \circ \Psi^{\phi} = \varphi \circ d\phi$. If ϕ is φ-invariant of constant rank then Ψ becomes of constant rank and therefore it becomes a (pseudo) f-structure on M which we call it *ϕ-associated (pseudo) f-structure.* In that case, we have that $\mathcal{H}_{\phi} = \mathcal{D}_{\Psi}$, and therefore we shall be using the letters $\mathcal{H}$ and D interchangeably for the same bundle. Setting $m = rank(\mathcal{H})$ and $s = rank(\mathcal{K})$ we see that (M^{m+s}, Ψ) becomes a (pseudo) f-manifold. Thus, every φ-invariant map of constant rank is (Ψ, φ)-holomorphic as a map $\phi : (M, \Psi^{\phi}) \to (N, \varphi)$ between (pseudo) f-manifolds. However, this ϕ-associated (pseudo) f-structure Ψ need not to be compatible with the prescribed metric g and therefore the triple (M^{m+s}, g, Ψ) need not be a metric (para) f-manifold.

Definition 1.2. *A map $\phi : (M, g) \to (N, h, \varphi)$ of a (semi) Riemannian manifold into a metric (para) f-manifold is said to be*

a°) *horizontally weakly conformal if g is nondegenerate on $K_{\phi}(p)$ for any $p \in M$ and $d\phi$ is surjective satisfying*

$$h(d\phi(X), d\phi(Y)) = \lambda g(X, Y) \quad \forall\ X, Y \in \mathcal{H}$$

for some smooth function $\lambda : M \to \mathbb{R}$.

b°) *(g, φ)-pseudo horizontally weakly conformal (or simply pseudo horizontally weakly conformal when no confusion arises) if ϕ is φ-invariant and ϕ-associated pseudo f-structure Ψ^{ϕ} is (hyperbolic) compatible with the metric g.*

Remark. Note that for a horizontally weakly conformal map $\phi : (M, g) \to (N, h, \varphi)$ we have

i) $\forall X, Y \in \mathcal{H}$,

$$\begin{aligned} \lambda g(\Psi(X), \Psi(Y)) &= h(d\phi \circ \Psi(X),\ d\phi \circ \Psi(Y)) = h(\varphi \circ d\phi(X),\ \varphi \circ d\phi(Y)) \\ &= r h(d\phi(X), d\phi(Y)) = \lambda r g(X, Y) \end{aligned}$$

that is, $\Psi = \Psi^{\phi}$ is (hyperbolic) compatible with the metric g,

ii) the surjectiveness of ϕ implies its φ-invariance.

Thus, from i) and ii) we conclude that every horizontally weakly conformal map $\phi : (M, g) \to (N, h, \varphi)$ is also pseudo horizontally weakly conformal.

2. Constancy of certain maps

Let $\phi : M \to (N, \varphi)$ be a smooth map of a smooth manifold into a (pseudo) f-manifold.

Definition 2.1. *The pair (ϕ, N) is said to satisfy condition $(\mathcal{A})$ if $\forall p \in M$ with $q = \phi(p)$, there exist a local section $X \in (d\phi(TM)) \subseteq (TN)$ such that*

$$[X, \varphi X](q) \notin D_{\varphi}(q).$$

Note that one has wide variety of choices of pairs (ϕ, N) by which the condition $(\mathcal{A})$ is satisfied. We give the following examples. Let N be a globally framed (hyperbolic) metric (para) f-manifold. Consider the following cases:

1°) Let $N = (N^{2n+\ell}, h, \varphi, \xi_j, \eta^j)$ be an almost (para) $\mathcal{S}$-manifold or in particular a (para) contact metric manifold. Observe that for any $j \in \{1, \ldots, \ell\}$, $\ell \geq 1$ and $\forall q \in N$ we have

$$\eta^j([X, \varphi X]) = -d\eta^j(X, \varphi X) = -\Omega(X, \varphi X) = rh(X, X),$$

$\forall X \in D_\varphi$, where $d\eta^j(X, Y) = \Omega(X, Y) = h(X, \varphi Y)$. So $\eta^j([X, \varphi X](q)) \neq 0$ and therefore $[X, \varphi X](q) \notin D_\varphi$, $\forall X \in D_\varphi$ with $h(X(q), X(q)) \neq 0$. Thus for $\phi : M \to N$ with $d\phi(TM) \subseteq D_\varphi$ the pair (ϕ, N) satisfies $(\mathcal{A})$.

2°) Let $N = (N^{2n+1}, h, \varphi, \xi, \eta)$ be a nearly (para) Sasakian manifold. Since N is nearly (para) Sasakian, we have $\forall X \in D_\varphi$,

$$(\nabla_X \varphi)X = h(X, X)\xi - r\eta(X)X = h(X, X)\xi.$$

On the other hand, by (2.1) we have $\forall X \in D_\varphi$,

$$\begin{aligned} \eta([X, \varphi X]) &= \eta(\nabla_X(\varphi X) + r\nabla_{(\varphi X)}(\varphi^2 X)) = \eta((\nabla_X \varphi)X + r(\nabla_{(\varphi X)}\varphi)(\varphi X)) \\ &= \eta(h(X, X)\xi + rh(\varphi X, \varphi X)\xi) = \eta(2h(X, X)\xi) = 2h(X, X). \end{aligned}$$

Thus by the same argument as above, for the pair (ϕ, N) with $d\phi(TM) \subseteq D_\varphi$ the condition $(\mathcal{A})$ follows.

3°) Let $N = (N^{2n+1}, h, \varphi, \xi, \eta)$ be an almost trans-Sasakian manifold of type (α, β) with $\alpha(q) \neq 0$, $\forall q \in N$. Then since N is trans-Sasakian, we have

$$\begin{aligned} \eta([X, \varphi X]) &= -d\eta(X, \varphi X) = -\alpha\Omega(X, \varphi X) + \tfrac{1}{n}(\eta \wedge \varphi^*(\delta\Omega))(X, \varphi X) \\ &= -\alpha\Omega(X, \varphi X) = \alpha h(X, X) \end{aligned}$$

$\forall X \in D_\varphi$. Thus, again for the pair (ϕ, N) with $d\phi(TM) \subseteq D_\varphi$, the condition $(\mathcal{A})$ follows.

Remark. Observe that for an almost (para) $\mathcal{C}$-manifold, nearly (para) cosymplectic and quasi K-(para) cosymplectic manifolds N the condition that $\mathcal{S}(X, X) \in D$, $\forall X \in D$ trivially holds. Therefore for the pair (ϕ, N) with $d\phi(TM) \subseteq D_\varphi$, the condition $(\mathcal{A})$ cannot hold.

Theorem 2.2. *Let $\phi : M \to (N^{k+\ell}, \varphi)$ be a φ-invariant map from an arbitrary smooth manifold into a (pseudo) f-manifold (with $\ell = rank(\mathcal{V}_\varphi) \neq 0$), such that the pair (ϕ, N) satisfies the condition $(\mathcal{A})$. Then ϕ is constant, that is $\overline{M} = \{p \in M : d\phi_p \neq 0\}$ is empty.*

Proof. Suppose $\overline{M}$ is not empty, then $d\phi_{p_0} \neq 0$ for some $p_0 \in M$. Set $q_0 = \phi(p_0)$ and $\mathcal{H} = \mathcal{K}^\perp$ with respect to any chosen Riemannian metric on M. By the hypothesis there is a (local) section X of $(d\phi(TM)) \subseteq D_\varphi \subseteq (TN)$ with $[X, \varphi X](q_0) \notin D_\varphi(q_0)$. Let Y be a local section of $\mathcal{H}$ with $d\phi(Y) = X$. Recalling the endomorphism $\Psi = \Psi^{\phi}{}_p : T_pM \to T_pM$ defined by (1.1), (note that then we have $\varphi \circ d\phi = d\phi \circ \Psi^{\phi}$), observe that

$$Z := [X,\ \varphi X] = [d\phi(Y),\, \varphi d\phi(Y)] = d\phi([Y, \Psi Y])$$

which shows that $Z_{q_o} \in d\phi(T_{p_0}M) \subseteq D_{\varphi}(q_o)$. This is a contradiction with the choice of X. This completes the proof.

Remark. In the above theorem

i) it is essential that $rank(\mathcal{V}_{\varphi}) = \ell > 0$ as the conditions $(\mathcal{A})$ cannot possibly hold for any (pseudo) f-manifold N with $\ell = 0$,

ii) also note that we do not impose any condition on M,

iii) pseudo f-manifold N need not be a para f-manifold.

Corollary 2.3. *Let $\phi : (M, \varphi^M) \to (N, \varphi^N)$ be a $\pm(\varphi^M, \varphi^N)$-holomorphic map between (pseudo) f-manifolds with $d\phi(TM) \subseteq D^N$ and suppose that the condition $(\mathcal{A})$ holds for the pair (ϕ, N). Then ϕ is constant.*

Proof. Observe that $\pm(\varphi^M, \varphi^N)$-holomorphicity and the assumption that $d\phi(TM) \subseteq D^N$ imply that ϕ is φ-invariant. So by Theorem 2.2, ϕ is constant.

Corollary 2.4. *Let $(M, J) \to (N, \varphi)$ be a $\pm(J, \varphi)$-holomorphic map from an almost (para) complex manifold into a (pseudo) f-manifold such that the condition $(\mathcal{A})$ holds for the pair (ϕ, N). Then ϕ is constant.*

Proof. Observe that since J is an almost (para) complex structure, $\pm(J, \varphi)$-holomorphicity gives that $d\phi(TM) \subseteq D^N$. Then the result follows from Corollary 2.3.

We say that a metric (para) f-manifold N is in $\star$-*category* if N is either nearly (para) Sasakian or trans-Sasakian of type (α, β) with $\alpha(q) \neq 0,\ \forall\ q \in \phi(M)$ or an almost (para) $\mathcal{S}$-manifold.

Corollary 2.5. *Let $\phi : (M, J) \to (N^{2n+\ell}, h, \varphi, \xi_j, \eta^j)$ be a $\pm(J, \varphi)$-holomorphic map from an almost (para) complex manifold into a manifold which is in $\star$-category such that, for the para cases, $d\phi(T_pM)$ is contained in neither $D^+(q)$ nor $D^-(q)$ $\forall\ p \in M$, where $q = d\phi(p)$. Then ϕ is constant.*

Proof. Since the target manifold is in $\star$-category, under the hypothesis, the condition $(\mathcal{A})$ holds for the pair (ϕ, N). Thus the result follows from Corollary 2.4.

In particular, when $(N^{2n+\ell}; h, \varphi, \xi_j, \eta^j)$ is an almost $\mathcal{S}$-manifold, Corollary 2.5 gives immediately the result in ([5], Theorem 5.2 and therefore Theorem 5.1).

Corollary 2.6. *Let $\phi : (M, g) \to (N^{2n+\ell}; h, \varphi, \xi_j, \eta^j)$ be a (g, φ)-pseudo horizontally weakly conformal map from an arbitrary (semi) Riemannian manifold of any dimension $m \geq 2$ into a manifold which is in $\star$-category such that for the* *para cases $d\phi(T_pM)$ is contained in neither $D^+(q)$ nor $D^-(q)$ $\forall\ p \in M$, where $q = d\phi(p)$. Then ϕ is constant.*

Proof. Under the circumstances the (g, φ)-pseudo horizontal weak conformality of ϕ and N being in the $\star$-category imply that the hypothesis of Theorem 2.2 is satisfied. Thus the result follows immediately.

Following the terminology in [11], a strongly pseudoconvex *CR*-manifold $(N^{2n+1}$ $\varphi_{_D}, D^N, \eta)$ with its Levi distribution D^N, of rank $2n$, 1-form η and positive definite Levi form $L(X,Y) = -d\eta\,(\varphi_{_D}X, Y)$, may be viewed as a contact metric manifold $(N^{2n+1}; h, \varphi, \xi, \eta)$ with $\varphi\,|_{_D} = \varphi_{_D}, h\,|_{_D} = L$ and η, its contact form, ξ, its characteristic vector field. Thus for such manifolds we have:

Corollary 2.7. *Let $\phi : M \to (N^{2n+1}; \varphi_{_D}, D^N, \eta)$ be a map into a strongly pseudoconvex almost CR-manifold*

i) *If there is an arbitrary almost complex structure J on M so that ϕ is a $\pm(J, \varphi)$-holomorphic, then ϕ is constant.*

ii) *If there is an arbitrary Riemannian metric g on M so that ϕ is a (g, φ)-pseudo horizontally weakly conformal, then ϕ is constant.*

This corollary recovers the result given in [11], Proposition 2.5. Note here that the condition imposed therein that M is semi-Kaehler is removed. We also include the case of ϕ being (J, φ)-antiholomorphic as well as (J, φ)-holomorphic.

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